



## Analysis of the Hankel Matrix in Embedding Using the Singular Spectrum Analysis (SSA) Method

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### ABSTRACT

Singular Spectrum Analysis (SSA) is an effective method of decomposition of time series for separating key components in data, such as trends, seasonality, and noise. This study aims to analyze the role of Hankel matrix in the SSA embedding process and how window length ( $L$ ) selection can affect the effectiveness of component separation in data time series. In this study, the data used includes public data that can be influenced by seasonal factors and unexpected events, such as natural disasters or regulatory changes. The research process begins with the data preprocessing stage, followed by the embedding stage to form a matrix used in decomposition with Singular Value Decomposition (SVD). To evaluate the similarity of separate components,  $w$ -correlation is used. The results show that the selection of optimal window lengths, in the range of  $N/4 < L < N/2$  is very important to maintain a balance between temporal information and matrix dimensions. With the right window selection, the embedding process in SSA can be more effective in separating the trending, seasonal, and noise components in the data pattern. By understanding the structure of the Hankel matrix and selecting the right parameters, the embedding process in SSA can be more effective in separating the components of the time series and preserving temporal information.

## **INTRODUCTION**

Time series analysis is a statistical method used to analyze data collected sequentially over time to find meaningful patterns or characteristics. (Box et al., 1973) explains that among the various time series analysis techniques, Singular Spectrum Analysis (SSA) has evolved as an accurate method due to its ability to decompose time series into interpretable components such as trends, seasonality, and noise. In time series analysis using SSA, the embedding process plays a very important role in the formation of the Hankel matrix, which subsequently becomes the main basis for spectral decomposition. In time series analysis using SSA, the embedding process plays a very important role in the formation of the Hankel matrix. This matrix becomes the main foundation for spectral decomposition. The uniqueness of Hankel's matrix structure allows for a more spectral informative representation of time series in matrix format. Therefore, understanding the structure and characteristics of Hankel matrices in the context of embedding SSA is key to improving the effectiveness of this method. The hallmark of the Hankel matrix allows for the representation of time series in a matrix format that is richer in spectral information. Therefore, understanding the structure and characteristics of the Hankel matrix in the context of embedding SSA is crucial to improving the effectiveness of this method. This understanding is to analyze the basic characteristics of the Hankel matrix in SSA embedding, discuss the influence of anti-diagonal properties on the decomposition process, and evaluate the importance of parameter selection in ensuring component separability. In its implementation, SSA relies heavily on an embedding process that uses Hankel's matrix as its mathematical basis. The structure of the Hankel matrix has a fundamental role in the transformation of univariate time series into trajectory matrices at the embedding stage of SSA (Zotov dan Shlemov, 2021). The structure of the Hankel matrix with its constant anti-diagonal characteristics has mathematical significance in time series transformations. The anti-diagonal structure of the Hankel matrix helps maintain the time sequence in the data, making the decomposition of components more accurate.

The Hankel matrix is a matrix with elements that have the same value along its anti-diagonal, which serves to preserve temporal information in time series data. This structure has a fundamental role in the transformation of univariate time series into trajectory matrices at the embedding stage of SSA. (Golyandina dan Zhigljavsky, 2013) has examined certain aspects of Hankel's matrix in SSA, particularly in relation to the properties of algebra in the context of component separability. Meanwhile, (Hassani dan Mahmoudvand, 2013) provides a new perspective on the implications of Hankel matrix structure on component reconstruction accuracy. However, there is still a gap in the theoretical understanding of the relationship between Hankel matrix characteristics and SSA decomposition quality. Improper selection of embedding parameters can result in a mixing effect between components, but does not provide a good theoretical basis for the selection criteria (Chen and Zhang, 2019). Optimization of embedding parameters requires an in-depth analysis of the

properties of the Hankel matrix, but does not provide a comprehensive mathematical pattern for this (Zhangt al., 2019).

The key parameter in the embedding process is the window length (L), which determines the dimensions of the Hangel matrix. Selecting the optimal window length is still a challenge because it directly affects the method's ability to separate the components of the time series (Golyandina et al., 2019). This suggests that the theoretical understanding of the relationship between the structure of the Hankel matrix and the quality of decomposition still requires further study (Gower et al., 2011).

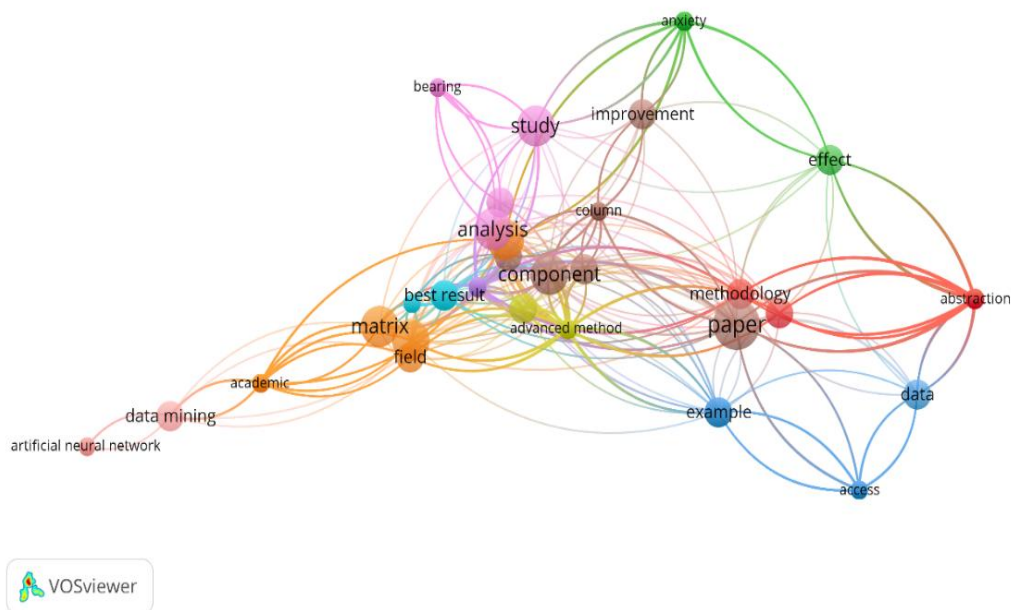
Based on this background, this study aims to mathematically analyze the properties of Hankel matrices in the context of embedding SSA. In particular, the study will analyze the relationship between the anti-diagonal structure of the Hankel matrix and the conditions of the separability of time-series components, develop theoretical criteria for the selection of optimal window lengths based on the characteristics of the Hankel matrix matrix, and evaluate the implications of the properties of the Hankel matrix on the decomposition quality of the SSA.

## LITERATURE REVIEW

This section discusses the visualization of bibliometric networks, Hankel's matrix, window length, and the orthogonality theorem.

### *Bibliometric Network Visualization (VOSviewer)*

VOSviewer is a software that allows its users to check bibliometric network maps for free (Yatscoff dan Hayter, 1983). VOSviewer has an advantage over other analysis programs because it uses text mining methods to find related phrase mapping combinations as well as creating grouping methods for data analysis VOSviewer has an advantage over other analysis programs because it uses text mining methods to find related phrase mapping combinations as well as creating grouping methods for data analysis (Saputro et al., 2023).



**Figure 1.** Visualization with Title and Abstract Constraints

Based on Figure 1, several clusters are formed which are marked with 8 different colors. The red cluster describes the majority of the research on methodology, the purple cluster describes the majority of the research on analysis, the blue cluster describes the majority of the research on data, the orange cluster describes the majority of the research on the matrix, and the brown cluster describes the majority of the research on the matrix component. Thus, there are still a few clusters that examine Hankel and SSA matrices.

- Hankel Matrix

The Hankel matrix is a matrix of size  $L \times K$  that has a special characteristic where each element on its anti-diagonal is of the same value. The Hankel matrix has a special structure in which each element on its anti-diagonal is of equal value, which plays an important role in the time series transformation of the SSA embedding process (Golyandina dan Zhigljavsky, 2013). The main advantage of the Hankel matrix in time series analysis (SSA) lies in its ability to maintain temporal relationships between elements. The concept of anti-diagonal invariants explains this, as it allows mapping of time-series elements into a more orderly structure. From a mathematical point of view, this provides the basis for a more stable approach to decomposition, particularly in separating the deterministic and stochastic components in a time series. Advanced matrix analysis techniques that provide a new perspective on the conditions of parability in SSA (Rodriguez and González, 2022). The nature of the Hankel matrix has direct implications for the effectiveness of the SSA algorithm. A comprehensive mathematical framework for the analysis of parallelability based on the structure of the Hankel matrix, with a special focus on the orthogonality conditions between components. The Hankel matrix (Li et al., 2022)(Rodriguez dan Martinez, 2021)  $H = (h_{ij})$  has the basic property  $h_{ij} = h_{uv}$  for  $i + j = u + v$ , which provides consistency of temporal information along the anti-diagonal. This structure is essential for the transformation of univariate time series into decomposable multivariate representations.

In the context of SSA, a Hankel matrix is formed during an embedding process that converts the univariate time series  $x = (x_1, \dots, x_n)$  into a trajectory matrix. For window length  $L$ , the Hankel  $H$  matrix can be written as

$$H = \begin{pmatrix} X_1 & X_2 & \dots & X_k \\ X_2 & X_3 & \dots & X_{k+1} \\ \vdots & \vdots & \ddots & \vdots \\ X_l & X_{l+1} & \dots & X_n \end{pmatrix} \quad (1)$$

where  $L$  is the selected window length,  $K = N - L + 1$  is the number of columns,  $N$  is the length of the original time series, and  $h_{ij} = x_{i+j-1}$  for  $1 \leq i \leq L$  and  $1 \leq j \leq K$ .

The numerical stability in SSA is greatly influenced by the structure of the Hankel matrix. In numerical analysis, the existence of extreme eigenvalues, both very small and very large, can lead to high sensitivity in the calculation of singular values, which in turn affects the decomposition results. Therefore, the application of a regularization approach or optimal parameter selection is very important to ensure stability in the use of SSA. The anti-diagonal structure of the Hankel matrix plays a crucial role in maintaining the numerical stability of the

SSA. The uniqueness of this structure not only affects the representation of data in higher-dimensional vector spaces, but also contributes to retaining temporal information contained in time series. In the context of linear algebra, Hankel matrices are often analyzed in conjunction with spectral decomposition to understand how the characteristics of singular values contribute to the separation of components. Further, numerical stability in Hankel matrix processing is highly dependent on the selection of the right embedding parameters. A small error in parameter selection can result in a mixing effect that can blur the boundary between deterministic and stochastic components. Therefore, the development of methods to determine optimal parameters is crucial in the implementation of a more accurate and stable SSA.

The structure of the Hankel H matrix in SSA has a special characteristic whereby each element of the  $f$  anti-diagonal  $k$  fulfills  $i + j = k + 1$ . The anti-diagonal structure of the Hankel matrix has a significant role in maintaining numerical stability in Singular Spectrum Analysis (SSA). The uniqueness of this structure not only affects the way data is represented in higher vector spaces, but also contributes to the preservation of temporal information contained in time series. In the study of linear algebra, Hankel matrices are often analyzed in relation to spectral decomposition to understand the contribution of singular value properties in component separation. One of the main advantages of the Hankel matrix in SSA is its ability to maintain temporal relationships between elements. This can be understood through the concept of anti-diagonal invariants that allow the mapping of elements in a time series into a more orderly structure. Mathematically, this provides the basis for a more stable decomposition approach, especially in separating the deterministic component from the stochastic component in a time series.

### ***Window Length and Compatibility***

The selection of window length ( $L$ ) is a crucial aspect that affects the effectiveness of SSA decomposition. According to (James, 2019), window The optimal length must meet two main criteria: large enough to capture patterns in the data and not large enough to avoid mixing effects. A theoretical approach based on matrix properties for the selection of optimal window lengths, taking into account aspects of dimensionality and preservation of temporal information (Kim dan Park, 2021). A more comprehensive theoretical framework for the analysis of SSA based on matrix properties (Moskvina dan Zhigljavsky, 2021). Matrix-based optimization approach for optimal window length selection (Park dan Kim, 2021). The parallelibility of the time series components depends on the orthogonality conditions affected by the structure of the Hankel matrix and the selection of window lengths.

The selection of window length affects the dimensions of the formed Hankel matrix. The selection of window length  $L$  has theoretical implications for the structure of the Hankel matrix formed. The  $N/4 < L < N/2$  boundaries are based on the inferior boundaries ( $N/4 < L$ ) which provide minimal dimensions to capture temporal patterns, provide ample space for spectral analysis, and the formation of a representative Hankel matrix. Superior limit ( $L < N/2$ ) that

optimizes decomposition without information redundancy, prevents information redundancy in the Hangel matrix, and minimizes mixing effects between components (Sanei dan Hassani, 2020) . If the L is too small, the information obtained from the embedding becomes limited, thus not allowing effective separation of components. Conversely, if the L is too large, excessive information redundancy occurs, which can lead to inaccuracies in the separation of the main signal and noise. Parameter selection also affects numerical stability in spectral decomposition. If the selected value is too small, the singular value of the Hankel matrix may not reflect the existing patterns in the data well, which may result in errors in the signal reconstruction. On the other hand, if the selected value is too large, the resulting singular value can contain a considerable amount of noise, which risks obscuring important information in the time series. Based on the matrix theory approach, the selection of L must also consider the singular value distribution of the Hankel matrix. The orthogonality properties of singular vectors in SVD play a key role in determining the extent to which a component can be separated. Thus, the selection of L must take into account the spectral characteristics of the Hankel matrix in order to maintain the decomposition quality. The spectral structure of the Hankel matrix can be analyzed through the decomposition of singular values written as

$$H = U\Sigma V^T \tag{2}$$

where  $U$  and  $V$  are matrix with orthonormal columns,  $\Sigma$  is a diagonal matrix with a singular value  $\sigma_i \geq 0$ , and the condition  $\sigma_i > \sigma_{i+1}$  indicates the separability of the components.

To measure the quality of the parapherability of the time series components  $F^{(1)}$  and  $F^{(2)}$ , the concept of w-orthogonality is used. The two components of the time series  $F^{(1)}$  and  $F^{(2)}$  are said to be w-orthogonal if the inner product of the column of the matrix of its trajectory is close to zero which is expressed as

$$\langle w_1, w_2 \rangle = 0 \tag{3}$$

where  $w_1$  and  $w_2$  are the columns of the trajectory matrix  $H^{(1)}$  and  $H^{(2)}$ . An inner product value that is close to zero indicates that the two components can be well separated.

The strength of the parability ( $v$ ) can be measured by stating as

$$v = \frac{\|U^T V\|}{\sqrt{(tr(U^T U)tr(V^T V))}} \tag{4}$$

where  $U$  and  $V$  are the trajectory matrices of different components,  $tr$  indicates the trace operator is the sum of the main diagonal elements of the matrix to provide a quantitative measure of how well the two components can be separated in the space of the Hankel matrix, and a value of  $V$  close to zero indicates good compatibility.

**Fundamental Theorem**

Hankel's matrix analysis in SSA has an important theorem basis. introduces the orthogonality theorem which states that the two components  $F^{(1)}$  and  $F^{(2)}$  of the time series  $F$  are said to be separated if the corresponding trajectory matrix has an inner product of zero. In the context of SSA, the additive decomposition (Hassani dan Mahmoudvand, 2013)  $F = F^{(1)} + F^{(2)}$  induces the decomposition of the Hankel matrix  $H = H^{(1)} + H^{(2)}$ , where  $H^{(k)}$  inherits the Hankel properties of the  $F^{(k)}$  component. The threshold  $\varepsilon$  represents the theoretical upper limit for weak coherbility and the theoretical lower limit for strong coherability in Hankel's matrix space. The weak and strong separability conditions are interrelated through the spectral structure of the Hankel matrix. The threshold  $\varepsilon$  of 0.1 was established based on a theoretical analysis of the balance between the sensitivity of component separation and the numerical stability of decomposition. This value is the theoretical upper limit for weak parability, where the two components are considered quite separate if the normalized inner product is less than this threshold. W-correlation  $\rho_{ij}$  provides a quantitative measure of orthogonality that is directly related to the separation capacity of components in Hankel's matrix space.

The orthogonality theorem states that the two components  $F^{(1)}$  and  $F^{(2)}$  are said to be weak parallelibility if they meet the stated as

$$\frac{\|X^{(1)}X^{(2)T}\|}{\|X^{(1)}\|\|X^{(2)}\|} < \varepsilon \tag{5}$$

where  $\varepsilon$  is the threshold value that determines the degree of separability, and  $X^{(k)}$  is the trajectory matrix of  $F^{(k)}$ .

For strong parallelivity, additional conditions are required at singular values (Chen dan Zhang, 2019) which is stated as

$$|\sigma_i^{(1)} - \sigma_j^{(2)}| > \delta \tag{6}$$

where all  $i$  and  $j$  are indices that show the different components of the SSA decomposition result, where  $\sigma_i^{(k)}$  is the  $i$  singular value of the  $k$  component, and  $\delta$  is the minimum threshold for the gap between singular values. A systematic approach to verifying the conditions of similarity involving the calculation of the inner product w-correlation (Golyandina dan Shlemov, 2015) which is stated as

$$\rho_{ij} = \frac{\langle \tilde{f}_i, \tilde{f}_j \rangle}{\|\tilde{f}_i\|\|\tilde{f}_j\|} \tag{7}$$

with  $\rho_{ij}$  is a measure of the weighted correlation between the two components of the reconstruction of the SSA, where  $\tilde{F}_i$  and  $\tilde{F}_j$  are the results of the reconstruction of different components.

The SSA not only serves as a tool for analyzing time series, but it also has relevance in the field of pure mathematics, especially in the context of spectral analysis and matrix decomposition. The Hankel matrix decomposition applied in SSA is often associated with eigen-value theory, which describes how the spectral information of a system can be broken down into smaller, more understandable components.

The relationship between SSA and other spectral methods illustrates the utilization of Hankel matrix decomposition in a variety of fields, including numerical optimization and multivariate data analysis. One of the interesting aspects is the application of SSA in dimension reduction, where the decomposition of singular values allows for a deeper understanding of the underlying data structure. In particular, SSA representations utilizing singular value decomposition (SVD) can be attributed to spectral transformations in orthogonal function analysis. This approach suggests that SSA is essentially a development of classical methods such as Karhunen-Loève transformations, which are widely applied in signal processing and data compression.

## RESULTS AND DISCUSSION

The theoretical analysis that has been carried out shows that the structure of the Hankel matrix in the embedding of SSA provides a solid basis for the separation of time series components. The anti-diagonal structure of the Hankel matrix not only preserves temporal information but also contributes to the effectiveness of spectral decomposition. The transformation of the  $X$  time series to the Hankel matrix  $H$  results in a mathematical structure with the following properties

### *Structure Anti-diagonal*

The properties of anti-diagonal structures are written as

$$h_{ij} = h_{uv} \quad (8)$$

with  $i + j = u + v$  produces an invariance that guarantees temporal preservation. This structure is fundamental for the spectral analysis of time-series components.

The anti-diagonal structure of the Hankel matrix has important implications in three aspects, namely, Invariance along the anti-diagonal results in the original time series temporal information being preserved, temporal order and dependencies remain unchanged, and the basis for the analysis of temporal components is established. A spectral decomposition of  $H$  that results in an orthonormal basis for the trajectory space ( $U$  column), a spectrum of singular values that reflects the structure of the components, and a coordinate transformation that preserves the temporal structure. Hankel's structure provides a theoretical basis for weak separability through w-correlation ( $p_{ij} < \epsilon$ ), strong separability through singular value gaps ( $|\sigma_i^{(1)} - \sigma_j^{(2)}| > \delta$ ), and the separation of components that preserve temporal structure.

### *Implications of Algebra*

The decomposition of the singular value of the Hankel matrix is expressed as

$$H = U\Sigma V^T \quad (9)$$

by generating an orthonormal base for the trajectory space and the singular value in  $\Sigma$  gives a measure of the contribution of each component.

The decomposition of the singular values in equation (3) results in a matrix of U and V whose columns form an orthonormal basis for the row spaces and columns of the Hangel matrix. The diagonal matrix  $\Sigma$  contains singular values in descending order, where the magnitude indicates the relative contribution of each component to the original time series. This spectral structure provides a separation of components based on the characteristics of their singular values.

The relationship between the anti-diagonal structure in equation (2) and spectral decomposition in equation (3) provides a theoretical basis for the analysis of separability. The orthogonality measured by w-correlation in equation (4) has a geometric interpretation in Hankel's matrix space, where the threshold of  $\varepsilon$  i.e. 0.1 represents the theoretical upper limit to indicate weak parallelivity and the maximum limit of the permissible correlation between the two components in order to be considered separate.

The condition of the separability of the time series components in SSA is closely related to the orthogonality properties of the trajectory matrix columns. For the two components  $F^{(1)}$  and  $F^{(2)}$ , w-orthogonal parallelibility is achieved by stating that

$$\rho_{ij} = \frac{|\langle \tilde{f}_i, \tilde{f}_j \rangle|}{\|\tilde{f}_i\| \|\tilde{f}_j\|} < \varepsilon \quad (10)$$

with a threshold of  $\varepsilon$ , which is 0.1 based on theoretical analysis.

Weak parallelability states that the two components  $F^{(1)}$  and  $F^{(2)}$  are weakly separated if they occur by stating as

$$\frac{\|X^{(1)}X^{(2)T}\|}{\|X^{(1)}\| \|X^{(2)}\|} < \varepsilon \quad (11)$$

If strong parallelability requires additional conditions on the singular value written as

$$|\sigma_i^{(1)} - \sigma_j^{(2)}| > \delta \quad (12)$$

with  $\delta$  is the minimum threshold for the singular value gap.

The selection of the L deep window length in the intervals  $N/4 < L < N/2$  optimizes the dimensionality of the passage space, the numerical conditions of decomposition, and the effectiveness of component separation. The selection of the optimal window length has a direct influence on the quality of component separation. The results of the theoretical study show that selection at certain intervals not only maintains a balance between temporal information and matrix dimensions but also guarantees numerical stability in the calculation of singular values. Improper parameter selection may result in a mixing effect that may reduce the effectiveness of the separation of the main components in the time series. The lower limit of  $N/4$  is necessary to ensure adequate temporal information coverage, while the upper limit of  $N/2$  prevents excessive duplication of information in the trajectory matrix. This study emphasizes that

*w*-correlation has a crucial role in assessing the degree of separation of time series components in the Hankel matrix space. Correlation values lower than a certain threshold indicate that the components extracted from the SSA have a good separation rate, resulting in a more accurate reconstruction. Verification of the condition of the separability is theoretically carried out through *w*-correlation analysis and evaluation of the singular value spectrum. The *w*-correlation matrix  $\rho_{ij}$  provides a quantitative measure of the degree of orthogonality between the reconstructed components. Singular value spectrum analysis includes checking the distribution of singular values and verifying the gap between the singular values of different components. The threshold  $\varepsilon$  of 0.1 has been theoretically proven to provide adequate criteria for proper separability.

SSA analysis shows that the data pattern is more stable with a significant seasonal uptrend. Compared to the moving average or ARRIMA method, SSA is better able to capture seasonal patterns without requiring the assumption of data stationarity. The results of the *w*-correlation test showed that the selection of the optimal window length contributed to increasing the interoperability of different data components. Thus, the SSA method can be used as a tool in detecting data trends and anticipating spikes that can affect the company's financial stability.

## CONCLUSIONS AND RECOMMENDATIONS

This study shows that the structure of the Hankel matrix in SSA embedding has an important role in the separation of time series components in data analysis. The selection of optimal window lengths in the range of  $N/4 < L < N/2$  allows for a balance between temporal information and matrix dimensions, thereby increasing the effectiveness of the method in identifying data patterns. By understanding the anti-diagonal structure of the Hankel matrix and verifying the separability conditions via *w*-correlation, the embedding process in SSA can be optimized for a wide range of practical applications in enterprise industries.

## ADVANCED RESEARCH

For further research, further exploration of the combination of SSA with machine learning techniques is recommended to improve the accuracy of data pattern predictions.

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