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A Model of Teaching Metacognition in Solving Mathematical Word Problems

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ABSTRACT

This qualitative study aimed to propose a model for teaching metacognition in solving mathematical word problems, utilizing a Multiple Case Study Method. The research explored how teachers employ metacognitive strategies, focusing on two components: knowledge of cognition and regulation of cognition. The findings suggest that metacognitive instructional techniques enhance students' mathematical knowledge and problem-solving abilities. Teachers who incorporate various metacognitive strategies help students develop their own learning skills and create conditions for meaningful learning. The study concludes that connecting metacognitive teaching approaches makes math problem-solving more significant. The proposed model allows teachers flexibility in applying strategies based on their circumstances and students' needs. Importantly, the research emphasizes that mathematics teachers must have a thorough understanding of mathematical concepts to effectively implement metacognitive methods.

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INTRODUCTION

Teaching metacognition is crucial to problem-solving because it enhances an individual's ability to understand, control, and manipulate their cognitive processes. Metacognition refers to the awareness and understanding of one's own thought processes, including the ability to monitor, plan, and regulate these processes. When applied to problem-solving, metacognition can significantly improve the efficiency and effectiveness of the problem-solving process.

"Cognition about cognition" or "knowing about knowing" is the definition of metacognition. It is the knowledge and awareness of one's own mental processes. It can also mean being aware of and knowledgeable about one's own thought processes and strategies, as well as having the capacity to deliberately consider and apply cognitive knowledge to alter such strategies and processes (Flavell, 1976).

Higher order thinking abilities known as metacognition entail more active control over the thought processes associated with learning. John Flavell, an American developmental psychologist, coined the term "metacognition" to describe this higher-level cognition (1976). The field of metacognition was first introduced by Flavell in the 1970s, when his research on children's metamemory centered on their awareness of and ability to regulate their memory processes. The question of how metacognition may or should be taught for students to obtain deeper insights into comprehending their reflections and perceptions to overcome their inadequacies in text comprehension became a hot topic in pedagogical circles during the 1980s and 90s.

Teachers and school psychologists have increasingly accepted the use of metacognition to assess students' skills and knowledge, even though scholars have long recognized its benefits (Baker, 2008). As a result, metacognition's function as a part of and a source of meaningful instruction has become much more popular in recent years. The question of whether, how, and why metacognition should be included in curricula or be a fundamental teaching technique has been discussed. Several empirical research (Haiduc, 2011; Cubukcu, 2008; Hall, 2004) demonstrate how important it is to educate metacognition to develop dexterity in these skills.

To maximize the benefits of metacognition strategies, students must organize their skills and be aware that they are utilizing metacognition, whether consciously or unconsciously. In the long run, this is the process that makes kids self-directed learners. According to Abdullah (2001), self-directed learners who master metacognition techniques don't require supervision. They possess the capacity to direct and regulate their thoughts, as well as to keep them on course.

Metacognition is the understanding of one's own thought processes and how to control and monitor them while performing a task (Goos, et al., 2000). According to Wells (2009) and Iwai (2011), metacognition is the process of thinking within the mind or thinking about thinking. The regulation of cognition and knowledge of cognition are two important characteristics that are highlighted in this concept. Declarative, procedural, and conditional knowledge are all components of knowledge about cognition (Schraw, 1998). Declarative knowledge is information on a person's identity, coping mechanisms, and the variables affecting their performance. Understanding procedure is the awareness of how to perform tasks. It involves being aware of which technique to employ. Knowing when and why to use cognitive tasks is known as conditional knowledge. It enables pupils to modify their knowledge in response to evolving circumstances.

Metacognitive regulation including organizing, observing, managing, and assessing cognitive processes are part of the regulation of cognition (Ader, 2019). Planning is selecting the best course of action for the work, whereas monitoring is being conscious of one's own actions while performing a task. Reviewing the entire process is part of control and evaluation (Schraw & Moshman, 1995).

In a community of learners, educators stress the value of an active and constructive process of sense-making, learning, and problem-solving (De Corte et al. 2011). Metacognitive skills have been incorporated into mathematics education as a necessary component by several nations (Ader, 2019).

Teachers' ought to provide opportunities for thinking and problem-solving, given the constant modifications made to mathematics curricula (Ader, 2019). In addition to assisting students in understanding the when, why, where, and how of applying their own knowledge to solve issues successfully, metacognitive skills are crucial for monitoring and regulating cognitive processes (Chan & Mansoor, 2007). Conversely, pupils who lack the ability to identify their own mistakes when solving problems, keep track of their progress, apply suitable techniques, or justify their answers are not proficient in mathematics (Carlson & Bloom, 2005; Lucangeli & Cabrele, 2006).

Consequently, one of the most important aspects of problem-solving is metacognition (Lester, 1994). Problem-solving is more than just putting methods into practice; it also requires metacognitive abilities in addition to cognitive strategies. Metacognitive skills are linked to problem-solving (Ader, 2013). Pupils possessing these skills can evaluate a problem's plausibility, assess how well strategies and solutions match, ascertain the accuracy of the solution, and identify their own mistakes. However, because metacognitive skills are hard to observe and analyze, math educators haven't investigated this topic enough.

According to Ozturk et al. (2020), the factors influencing problem-solving abilities can be divided into two groups: cognitive and affective skills. According to Belet and Yasar (2007), the cognitive variable relates to reading comprehension abilities such as determining the text's primary and supporting concepts, seeing cause-and-effect relationships, and deducing the meanings of any new terms. It will be difficult for pupils who don't think they can solve problems to solve them because they won't spend as much time on them. Self-efficacy perception and other affective abilities are important for problem solving (Hoffman & Schraw, 2009). In short, problem-solving abilities are influenced jointly by cognitive (metacognition, reading comprehension ability, intelligence, demand for cognition) and affective (mathematics self-efficacy, attitude, anxiety, beliefs, mathematics interest) capabilities (Ozturk et al., 2020).

Numerous research has examined the role that metacognition plays in problem-solving effectiveness (Artzt et al., 1992; Pennequin et al., 2010; Swanson, 1990). For example, Kuhn (2000) highlighted the impact of early metacognition development on higher order thinking processes since it improved cognitive abilities. Pupils did better on the assignments including problem solving and mathematics learning (Mevarech & Fridkin, 2006).

According to Eggen and Kauchak (2001), successful students are those who recognize when to behave strategically and when not to, since conscious learning is more effective. Students who use metacognition are better able to manage and complete the steps involved in problem-solving (Sevgi & Cagliköse, 2019).

To solve problems, one must consciously follow, regulate, and control the problem-solving steps in addition to carrying them out. Consequently, a crucial step in the problem-solving process is metacognition (Schoenfeld, 2005). While several studies (Kitsansas, 2002; Rysz, 2004) have demonstrated a favorable correlation between metacognition and mathematical success, other investigations (Ader, 2004; Hong & Peng, 2008; Kuyper et al., 2000) have not confirmed these results. This paradox also highlights the need for more investigation into the connection between teaching mathematics and metacognition.

In summary, teaching metacognition equips individuals with the skills and mindset needed to approach problem-solving in a systematic, reflective, and adaptive manner. This not only improves their problem-solving abilities but also contributes to lifelong learning and intellectual growth.

LITERATURE REVIEW

Metacognition is teachable – this is according to several researchers who offer empirical evidence on the “teachability” of metacognitive skills (Akyol & Garrison, 2011; Flavell, 1979; Leo et al., 2009; Lester, 1982; Schoenfeld, 1987; Verschaffel, 1999). The importance of regulation of cognition in teaching was emphasized in the study of Iiskala et al. (2004), Kramarski (2008), Leo et al. (2009), and Lester (1982). While the significance of knowledge of cognition in teaching was studied by Akyol and Garrison (2011), Anghileri (2006), Hashell (2001), Karpicke (2012), Mestre (2002), Meyer (2002), Mintres, et al. (2005), Philipp et al. (2007), Thames and Ball (2010), Terry (2006), and Watson and Madison (2006).

In terms of regulation of cognition, Schoenfeld (1987) made one of the early attempts to highlight the importance of metacognition for mathematics education. His study revealed that metacognition has the potential to increase the meaningfulness of classroom learning for students, and the best way to promote metacognition is to create a community of mathematics. Specifically, such mathematics culture means that students learn to think of mathematics as an integral part of their daily lives, helping them interact throughout different contexts between mathematical concepts.

In the study, Iiskala et al. (2004), observed that metacognitive instruction provides a cognitive regulation process namely self- regulation. Self-regulation is an intrapersonal regulation that pertains to the concept of monitoring and control of individual performance.

Foreign studies on problem solving for learners in the context of mathematics education have shown that learners have performed well in tasks if they have metacognitive skills (Kramarski, 2008; Leo et al., 2009). For example, in the study of Lester (1982), he decided to include metacognition in his list of questions, since he firmly believed that a person's knowledge of his own cognitions before, during, and after a problem-solving phase, as well as his ability to maintain executive control in the context of monitoring and self-regulation, should have a significant impact on successful mathematical problem-solving. Verschaffel (1999) also found out that in the process of mathematical problem solving, metacognition is important.

As to knowledge of cognition, Akyol and Garrison (2011) test students' metacognitive knowledge on how they will demonstrate it through on-line learning context. Researchers selected 3 weeks of online discussions (1st, 5th, and 9th) to evaluate respondents' metacognition and collected data of the respondents for the knowledge about cognition and regulation of cognition. Through observations, researchers found out that while knowledge of cognition decreased in time and regulation of cognition increased.

Similarly, it was found that teachers who have a metacognitive knowledge in mathematics teaching will eventually be able to represent conceptual ideas in instruction. Hence, it was a belief that decisions of teachers greatly rely on their conceptual understanding in mathematics knowledge. Therefore, mathematical models, concepts, and representations will be easy to communicate if a teacher holds a deeper mathematics pedagogical knowledge (Philipp et al., 2007). Thus, subsequent positive influence on instructional techniques advocates for a deep understanding of the subject matter knowledge (Thames & Ball, 2010).

As mentioned by Anghileri (2006) and Watson and Mason (2006), teachers now are taking care in choosing the right contexts that cannot distract students from the mathematical purpose tasks. Keeping mathematical subject connections and goals as short as possible can support those learners who are just focusing on some context issues at the expense of mathematics.

Additionally, Mayer (2002) and Mestre (2002) said that it is an important objective that a learner will use what they have learned, same goes in development of knowledge. Through transferring of knowledge, individuals can see that what they have learned is useful (Haskell, 2001; Mayer, 2002). Learners learn to use it to solve their problems in life and create new learning. To achieve meaningful learning, learners must receive explicit instructions or applicable ideas from long haul memory and elaboration, separation, and combination of those ideas to sort out cognitive structure (Mintzes et al., 2005; Karpicke, 2012; Terry, 2006).

In the end, many mathematics teachers can apply the rules and procedures in teaching mathematics but lack conceptual knowledge and skills in metacognition to move into a deep understanding of concepts and theory (Conference Board of the Mathematical Sciences [CBMS], 2012). Therefore, studying the metacognition for teachers' understandings on the way they act, challenges they encounter in doing it, and their metacognitive knowledge and regulation are important constructs. Gaining the knowledge of metacognition would likely help teachers to plan programs and developmental activities among learners.

Statement of Purpose

This qualitative study aimed to propose a model of teaching metacognition in solving mathematical word problems. Specifically, this study sought answers to the statement in pursuit of academic contributions in the area of mathematics education:

1. Describe how teachers teach learners' mathematical knowledge and skills in solving word problems in terms of the following components of metacognition:
   a. Knowledge of cognition; and
   b. Regulation of cognition.

METHODOLOGY

This study employed a multiple case study approach to investigate metacognitive teaching strategies for mathematical word problems. The research aimed to construct a paradigm for teaching metacognition by analyzing the practices of master teachers. A minimum of 12 Grade 10 Mathematics master teachers from different school clusters in the Division of Pampanga were selected as participants, using a combination of criterion-based and heterogeneous purposeful sampling. The selection criteria included factors such as educational background, work experience, and scores on a metacognitive skills inventory.

The study utilized four main instruments for data collection: an initial survey questionnaire, a virtual classroom observation guide, an online follow-up interview guide, and journal writing. The initial survey, administered via Google Forms, helped select participants and assess their understanding of metacognition. Virtual classroom observations were conducted to capture teaching practices, while semi-structured follow-up interviews explored teachers' conceptual understanding and classroom approaches. The researcher also maintained a reflective journal throughout the process.

Data analysis followed a grounded theory approach, employing a constant comparative method. The process involved three main phases: transcription of interviews and observation notes, initial coding, and the creation of axial and selective codes to determine themes. Microsoft Excel was used to organize the coding process. The analysis integrated data from multiple sources, including teacher interviews, classroom observations, and learner interviews, to triangulate findings and develop a comprehensive understanding of metacognitive teaching strategies in mathematics. This approach allowed the researcher to identify patterns and consistencies across participants' responses and practices.

RESEARCH RESULT AND DISCUSSION

Metacognitive Instruction in Solving Word Problems

From the group of master teachers, metacognitive techniques used in teaching solving word problems were categorized. Table 1 shows the overview of metacognitive strategies in teaching used during virtual classroom observations. The themes presented in the table summed up the teachers' metacognitive strategies in solving word problems using the two metacognitive components as framework of analysis namely: knowledge of cognition (declarative, procedural, conditional) and regulation of cognition (planning, monitoring, and evaluating).

From these two components of metacognition, several metacognitive strategies were identified from the gathered data. Along with knowledge of cognition, declarative knowledge manifests understanding and identifying concepts through reading and restating the problem and recalling and identifying the required information. Procedural knowledge revealed teaching by example through organizing solutions, solving part by part, and using parallel problems while generating questions from conditional knowledge through teaching similar problems and sharing of process.

As to regulation of cognition, planning regulation elaborates making representation by organizing problem specifics, figure illustration, and identifying patterns Moreover, monitoring regulation displays comparing and verifying solutions from comparing with similar problems, using a wrapper, reviewing solutions from time to time, and using self-talk. Finally, evaluating regulation manifests looking back strategy through checking the process, debriefing practices, and providing conclusions.

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Table 1. rview of metacognitive strategies in teaching used during virtual classroom observations

Metacognitive Components	**Types of Metacognition**	**Metacognitive Strategies**	**Elaboration of Metacognitive Strategies**
Knowledge of Cognition	Declarative Knowledge	● Understanding and Identifying Concepts	● Reading and Restating the Problem ● Recalling and Identifying the Required Information
	Procedural Knowledge	● Teaching by Example	● Organizing Solutions ● Solving Part by Part ● Using Parallel Problems
	Conditional Knowledge	● Generating Questions	● Teaching Similar

			Problems • Sharing of Process
Regulation of Cognition	Planning Regulation	• Making Representation	• Organizing Problem Specifics • Illustrating the Figure • Identifying Patterns
	Monitoring Regulation	• Comparing and Verifying Solutions	• Comparing with Similar Problems • Using a Wrapper • Reviewing Solution from Time to Time • Using Self-talk
	Evaluating Regulation	• Looking Back	• Checking of Process • Debriefing Practices • Providing Conclusions

The succeeding sections provide the different statements and reasons from the metacognitive strategies used by the selected master teachers in teaching word problems.

Teaching Knowledge of Cognition

Knowledge of cognition using its three metacognitive factors (declarative, procedural, and conditional) emerged through a specific metacognitive strategies such as understanding and identifying concepts, teaching by example, and generating questions which include different strategies that elaborate concepts such as: reading and restating the problem, recalling and identifying the required information, organizing solutions, solving part by part, using parallel problems, teaching similar problems, and sharing of process.

Teaching declarative knowledge pertains to a process of metacognition involving conceptions, individual cognitive goals, personal abilities, and beliefs on task structures (Schraw *et al.*, 2006). Through one episode of teaching, the teacher's coded responses were directed towards a metacognitive strategy: Understanding and Identifying Concepts. Understanding and identifying concepts are elaborated through specific metacognitive strategies such as: read and reread the problem, restate the task into their own words, recall and select needed resources by using previously acquired knowledge, concepts, or theories, and identify the required information needed to solve the given word problem.

Understanding and identifying concepts or factual knowledge involved in solving word problems during the online instruction include specific strategies that elaborate the concept such as: reading aloud and translation techniques that emanated students to see what was on the problem, to model the right pronunciation of mathematical expression, and to understand the given task. This also required a type of information to get a clear idea of the problem, and the data needed for solving it. Hence, chances among students to select applicable sources and identify the required information for solving the word problem were considered.

Reading and restating the problem as a technique requires translation of concepts based on the understanding of the learner towards the problem by underlining or highlighting key terms, clarifying learners' terminologies, and unlocking difficulties. One episode that merited further conversation was when students' understanding surfaced during the virtual classroom discussion.

From the data on procedural knowledge, teaching by example emerged as the major metacognitive strategy. This strategy was elaborated by specific metacognitive strategies such as: organizing solutions, solving part by part, and using parallel problems.

Teaching by example helps learners on how to solve the problem through the help of their teacher. This includes specific strategies that elaborate concepts by explaining the procedure of the word problem by example. This assists learners to learn how to process solving word problems by organizing the solution, solving the problem part by part, or using parallel problems to master the process.

Organizing solutions elaborated metacognitive strategy supports learners to learn on how to organize their solution logically and coherently through the proper guidance of their mathematics teacher. Organizing the solution coherently and logically in this study came from teaching sequential and standard procedures. This offered opportunities among learners to organize and manage their solving process.

Solving part by part elaborated metacognitive strategy assists learners on how to solve a word problem part by part through the guidance of their teacher. Solving part by part to the extent of this study elaborate specific concepts such as breaking down the lengthy problems into parts.

And using parallel problems elaborated metacognitive strategy helps learners on how to solve problems by remembering and understanding parallel problems to develop their mastery. Mastering the process includes specific strategies that elaborate concepts such as: giving consistent practices and parallel problems and allowing mistakes and errors. These would help learners to remember the process, master the procedure, and develop mathematical skills.

Conclusively, teachers help learners to learn how to write the solutions in each word problem by demonstrating the procedure, organizing solutions, solving the process part by part, and even generating solutions. All the participants in the study used to teach the process of solving problem's part by part but less were given to teach the learners on how to think of alternative processes in solving word problems. In this context, the teachers' procedural knowledge shows their knowledge on teaching thinking processes. The activities leading to an individual's ability to organize solutions and solve the problem by parts are entirely procedural. This process of metacognitive knowledge reflects on the process or individual strategies in solving particular problems. Individuals with procedural knowledge are composed of a repertoire of strategies in thinking and learning (Schraw *et al.*, 2006).

From the data on conditional knowledge, generating questions emerged as the major metacognitive strategy. This strategy was elaborated by specific metacognitive strategies such as: teaching similar problems and sharing processes.

Generating questions as a metacognitive strategy was elaborated through specific concepts such as: understanding the procedure in the problem, be familiar with the lesson, generate questions as to when and why to use such technique, and to see small details of errors in his own solution. In this direction, the discussion was focused, thus, according to the specific metacognitive strategies namely: teaching similar problems and sharing of process.

Teaching similar problems elaborated metacognitive strategy helps students to understand their thinking process on how to generate questions on the problem through the guidance of their teacher. This includes specific strategies that elaborate concepts like teaching parallel problems to develop familiarity. This opened opportunities among learners to master the procedure, be familiar with the lesson, and generate questions as to when and why to use such procedures.

Sharing of process elaborated metacognitive strategy helps learners to understand and clarify their thinking process by means of sharing it with the class through the proper guidance of the teacher. This includes specific strategies that elaborate concepts such as: sharing of understanding and solution in the class, which helped them to see small details of error in their own solution.

Teaching Regulation of Cognition

Regulation of cognition using its three metacognitive factors (planning, monitoring, and evaluating) emerged through a specific metacognitive strategies such as making representation, comparing and verifying solutions, and looking back which include different strategies that elaborate concepts such as organizing problem specifics, illustrating the figure, identifying patterns, comparing with similar problems, using a wrapper, reviewing solution from time to time, using self-talk, checking of process, debriefing of practices, and providing conclusions.

From the data on planning, making representations emerged as the major metacognitive strategy. This strategy was elaborated by specific metacognitive strategies such as: organizing problem specifics, illustrating the figure, and identifying patterns.

Making representations from a given word problem include specific metacognitive strategies that elaborate concepts such as: organizing problem specifics, illustrating the figure, and identifying patterns. These elaborated metacognitive strategies help learners learn how to organize problem specifics by writing the necessary steps or data, how to illustrate the problem through visualization, drawing, or graphing, and how to identify patterns that involve identification of concepts and familiarity.

Organizing problem specifics elaborated metacognitive strategy helps learners on how to manage the details in the problem. This constituted proper organization of data, writing of steps, or sequencing. This gave opportunities among learners to prevent confusion thus leading them to solve the task correctly.

Illustrating the figure elaborated metacognitive strategy helps learners on how to illustrate the problem through the guidance of their mathematics teacher. Figure's illustration in this study elaborates specific concepts such as: drawing a picture, visualization of the word problem, and graphing. This would give chances among students to visualize the problem, see the requirements needed to solve the problem, determine the missing part of the task, to decide what formula was appropriate, to figure-out the context, and to see the bigger picture of the problem.

And identifying patterns elaborated metacognitive strategy helps students to look for patterns when solving problems through the guidance of the teacher. Looking for a pattern in a problem elaborate specific concepts such as: identification of specific ideas and familiarity. This would give opportunities to the students to see the concept and be familiar whenever he/she would encounter similar problems.

From the data on monitoring, comparing and verifying solutions emerged as the major metacognitive strategy. This strategy was elaborated by specific metacognitive strategies such as: comparing with similar problems, using a wrapper, reviewing solutions from time to time, and using self-talk.

Comparing and verifying solutions pertains to a skill to perceive similar problems through giving parallel problems and regular practice. This includes specific strategies that elaborate concepts such as: comparing with similar problems, using wrapper during discussion, reviewing solutions from time to time, and self-talk techniques.

Comparing with similar problem elaborated metacognitive strategy assists learners on how to compare the procedure of the given word problem to a parallel problem taught from the past. Comparing similar problems to the extent of this study elaborating familiarization of the student with the problem by giving two or more examples. This offered chances to the students to develop mastery about the procedure of the problem and to see if they were doing the right process.

The use of wrapper could be a learning habit that can be taught through monitoring and adapting strategies. This can be a self-monitoring behavior that can be used to activities that encourage metacognition. Wrapper is a technique in teaching done before the beginning of the lesson in which the teacher asks learners' viewpoints about the topic, write them on a piece of paper, and reveal them after the discussion. This technique gives learners immediate feedback about their perceptions that guarantee an effective result in developing monitoring skills.

Reviewing solution from time-to-time elaborated metacognitive strategy brought opportunities to the students on how to review their solution from time to time whenever they solved word problems through the help of their teacher. Reviewing students' solutions from time to time to the limitation of this study and based on the result of the observations include specific strategies that elaborate concepts such as: process questioning technique, looking back from the procedures, stopping from time to time to check the work, and understanding the problem first before taking another step. This opened opportunities among learners to monitor students' progress in solving the problem and create an awareness on reviewing their solutions.

Using self-talk and talk-aloud as an elaborated metacognitive strategy gave opportunities among learners to express their thoughts about their understanding of the problem and difficulties encountered during the solving process. T3 and T8 asserted that during the online class discussion, she allowed her students to talk aloud if they had an idea about the given problem. She gave them the opportunity to express their thoughts about the given word problem while other students were listening.

To sum up, teachers demonstrated two techniques from the given subthemes in developing learner's monitoring regulation of cognition. Master teachers gave opportunities to compare the solution of their learners to similar problems they had already encountered, used wrapper as a technique in improving learners' monitoring skills, allow learners to do self- talk and talk-aloud techniques to discuss their own understandings, and made sure that their learners review their process from time to time to assure that their solution was correct in congruence to the problem, given, and formula.

Although the realm of discussion leans on the learner as the thinker, it is but equitable to bring teachers on equal footing, since they are part and parcel of the community of learners. The regulatory monitoring was about mathematics teaching meeting the goals - the teachers' teaching of mathematics and ultimately, the checking of learners' comprehension. Gopinath (2014) affirmed Flavell's (1979) and Brown's (1980) monitoring as a component of regulatory mechanism where a person checks his/her own understanding to determine the effectiveness of an attempt to solve a task using that understanding. Thus, there is a developed switching strategy as one learns how to alter from one strategy to another.

From the data on evaluating, looking back emerged as the major metacognitive strategy. This strategy was elaborated by specific metacognitive strategies such as: checking of process, debriefing practices, providing conclusions.

Looking back to the solution after solving the problem was elaborated through specific metacognitive strategies such as: reviewing and checking the process, using debriefing practices, and providing conclusions. These helped learners to evaluate their own work, enhance their decision and judgement about the problem, and do self-reflection.

Checking of process elaborated metacognitive strategy helps learners on how to check the process they made in solving the word problem through the help of their teacher. Checking the process to the extent of this study include specific strategies that elaborate concepts such as: checking if the derived answer was correct through substitution method, working backward method, checking if all the conditions were correct, making generalization, and revising the solution if the estimated answer was not achieved. This avenue opened opportunities to the students to evaluate their own work.

Debriefing practices elaborated metacognitive strategy helps students to learn how to provide feedback from their work after completing a task through the help of their teacher. Providing feedback after solving a problem to the extent of this study includes specific strategies that elaborate concepts such as: looking back to the solution, reviewing calculations and procedures, and using question-based strategies. This avenue opened opportunities to the students to enhance their decision and judgement about the problem, to review their calculation, to question their answer, and to think for the easiest way to solve the problem.

And providing a conclusion after solving the problem was so essential for the learners to understand the purpose of their computation. One teaching episode of T3 shows how she instructed her learners to provide a conclusion after solving the word problem.

The revised Bloom's taxonomy of educational objectives emphasizes that evaluation is a high-order thinking skill, as its values criteria-based judgment (Zawilinski, 2009). Teachers' ability on how to review and check the process, debrief practices, and make conclusions after solving the word problem is called the regulatory mechanism of evaluation. When one is explicit and conscious in controlling his/her own thoughts and action, as well as memories, these are overt conditions of evaluating knowledge (Rahman, 2011).

Metacognitive Teaching Strategies in Mathematics

There are two metacognitive components (knowledge of cognition and regulation of cognition) with three types for each metacognitive component that have investigated (*see figure 1*). For knowledge of cognition, there are three types of metacognitions - declarative, procedural, and conditional while three types under teachers' regulation of cognition that include planning, monitoring, and evaluating. One metacognitive strategy emerged from each type of metacognition. For declarative, there is one metacognitive strategy which is understanding and identifying concepts, while teaching by example for procedural and generating questions for conditional. For planning, there is one metacognitive strategy which is making representation while comparing and verifying solution for monitoring and looking back strategy for evaluating.

Understanding and Identifying Concepts	Teaching by Example	Generating Questions	Making Representation	Comparing and Verifying Solution	Looking Back
Declarative	**Procedural**	**Conditional**	**Planning**	**Monitoring**	**Evaluating**
Knowledge of Cognition			**Regulation of Cognition**		

Figure 1. Model of Metacognition in Mathematics Teaching

As to **knowledge of cognition**, the **teacher's declarative knowledge** that had emerged from teachers' practices was *understanding and identifying concepts* which include specific strategies that elaborate concepts such as reading and rereading the problem, restating the problem through their own words, recalling needed sources, and identifying the required information.

While **procedural knowledge** emerged *teaching by example* as a major metacognitive strategy with elaborated strategies such as organizing solutions, solving the problem part by part, and using parallel problems to master the process.

Moreover, a **teacher's conditional knowledge** is directed towards a metacognitive strategy - *generating questions*, which include specific strategies such as teaching similar problems and sharing of process.

And as to **regulation of cognition**, **teacher's planning regulation** includes *making representation* emerged as the major metacognitive strategies and elaborated through specific metacognitive strategies such as organizing problem specific, illustrating of figures, and identifying patterns.

Likewise, from the data on **teacher's monitoring regulation**, *comparing and verifying solution*s emerged as the major metacognitive strategy which directed towards a specific elaborated metacognitive strategy such as comparing with a similar problem, using a wrapper, reviewing their solution from time to time, and verifying their solution through self-talk or talk-aloud.

Finally, the **teacher's evaluating regulation** that had emerged from teachers' practices was *looking back* strategy as the major metacognitive strategy that include specific techniques such as the protocol of sharing to students on how to check the process in the problem, debriefing practices, and providing conclusions.

The rounded arrow in the figure above shows that the metacognitive teaching process is a parallel process which suggests that teachers' metacognitive strategies in teaching depends on the condition needed in the word problem. This means that it is not necessary to follow the sequential flow of the model above, but its purpose is to present the overarching strategies of the metacognitive teaching from declarative knowledge through evaluating regulation.

The metacognitive instructional process as shown in the figure created meaningful teaching that is parallel to students' experience called meaningful learning. Meaningfulness in teaching is an extract, a concept, from teachers who, by constant comparison, unceasingly stand by their learners and demonstrate the value of achievement and change for the better of the learner as well as offer academic and moral help to all kinds of learners.

CONCLUSIONS AND RECOMMENDATIONS

The research findings lead to the conclusion that students' mathematical knowledge and word problem-solving abilities are enhanced by metacognitive instructional techniques. In addition to improving students' mathematical performance, teachers who teach a variety of metacognitive strategies help students develop their metacognitive knowledge and skills, which gives them a stronger position to cultivate their own learning and create the conditions that support and encourage meaningful learning.

The use of metacognition to education opens new avenues for classifying the particular and sophisticated metacognitive techniques that math teachers employ, as well as the learning methods that students pick up from mathematical discourse. Making connections between these metacognitive techniques via the prism of instruction can encourage more fruitful and insightful classroom dialogue.

Math problem solving gains greater significance when metacognitive teaching approaches are connected and coherent. The process of choosing metacognitive techniques for instruction is not sequential or linear in the presented model. As a result, teachers are free to employ these strategies in accordance with their circumstances, subject-matter expertise, and the degree of mathematical knowledge and comprehension of their students. Most significantly, teachers of mathematics must have a thorough comprehension of mathematical concepts and procedures to apply metacognitive methods.

ADVANCED RESEARCH

The limitation of this study is its qualitative nature and relatively small sample size of 12 master teachers from a single school division. While this allowed for in-depth analysis, it limits the generalizability of the findings to other contexts or larger populations. The focus on Grade 10 Mathematics teachers also narrows the scope. Future research could expand to a larger, more diverse sample across multiple grade levels and geographic regions to provide a more comprehensive view of metacognitive teaching strategies in mathematics.

Another potential limitation is the reliance on virtual classroom observations and online interviews due to the pandemic context. While necessary, this may have impacted the depth and nuance of data collection compared to in-person observations and interactions. Additionally, the study focused primarily on teacher practices and did not directly measure student outcomes or improvements in metacognitive skills. Future studies could incorporate more direct assessments of student metacognition and problem-solving abilities before and after exposure to these teaching strategies to quantify their effectiveness.

Lastly, while the study developed a model of metacognitive teaching strategies, it did not test or validate this model empirically. Future research could operationalize the components of this model into measurable constructs and test their relationships and impacts on both teacher practices and student outcomes. Longitudinal studies examining how teachers develop and refine these metacognitive teaching strategies over time, as well as how students' metacognitive skills evolve in response, would provide valuable insights. Additionally, experimental studies comparing classes taught with explicit metacognitive instruction to control groups could help establish causal relationships between these teaching strategies and student outcomes in mathematical problem-solving.

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