

Effectiveness of Abstract – Visual – Concrete Approach in Enhancing Students’ Level of Conceptual Understanding and Mathematical Self-Efficacy

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ABSTRACT

This study was designed to transform mathematics instruction into a more engaging and practical learning experience, emphasizing real-world applications. The primary objective was to enhance conceptual understanding and mathematical self-efficacy among Grade 11 senior high school students in Statistics and Probability during the 2021-2022 academic year's second quarter. The findings demonstrated that mathematical self-efficacy positively correlates with conceptual understanding, with high-efficacy students showing greater confidence and positive attitudes toward mathematics. The Abstract-Visual Representation-Concrete (A-V-C) instructional approach proved effective in teaching Pearson product moment correlation and regression analysis. This method progresses from abstract concepts through behavioral modeling to exploratory concrete experiences, enabling students to develop comprehensive understanding through structured, hands-on learning.

INTRODUCTION

Mathematics literacy is essential for applying concepts, reasoning, and solving mathematical problems (NCTM, 1989), with problem-solving being particularly crucial for developing students' conceptual understanding and mathematical communication skills (Luke, 2012).

In Philippine education, mathematics remains a challenging subject for many students (Chua, 2006). This is evidenced by poor performance in national examinations, with the National Achievement Test (NAT) showing secondary level scores of just 46.38%. International assessments like TIMSS have consistently placed the Philippines near the bottom, ranking 39th out of 42 countries in 1995, third to last in 1999, and 41st among 42 participants in 2003 (Pangan, 2010).

Filipino learners significantly lag behind their Asian counterparts in mathematics mastery (Bilaso et al., 2008). This is demonstrated by poor performance in international competitions and national examinations, where two-thirds of schools score below average, failing to meet the Department of Education's 75% benchmark (Ogeña et al., 2008; Culaste, 2011).

The Concrete-Pictorial-Abstract Sequence Approach (CPA) offers a promising solution as an explicit instruction strategy that combines conceptual and procedural learning. This structured approach has proven particularly effective for passive learners and students with learning disabilities (Hudson et al., 2006).

This study focuses on implementing the concrete-visual representation-abstract (CVA) instructional model among grade eleven students at Telesforo and Natividad Alfonso High School during the 2021-2022 school year. The research employs a descriptive-experimental approach to analyze student achievement and perspectives, particularly focusing on correlation and regression analyses in statistics and probability.

LITERATURE REVIEW

Student achievement serves as a crucial indicator of educational quality, with numerous studies identifying various factors affecting mathematics performance. These studies emphasize the importance of appropriate tools, methods, and approaches in enhancing student learning outcomes (Mahanta, 2012).

Statistics and probability education in Philippine secondary schools is integrated into the mathematics curriculum. Batanero and Diaz (2011) highlight the importance of this integration, citing its practical applications in daily life, relevance to other disciplines, professional requirements, and role in decision-making under uncertainty.

Mathematics educators face significant challenges in teaching their subject effectively. Keeler and Steinhurst (2001) note that while extensive research documents difficulties in learning mathematics and uncertainty-related concepts, studies on potential solutions remain limited.

Research by Gomez (2010) on fourth-year high school students in Mabalacat, Pampanga, revealed average mathematical ability levels among participants. The study recommended regular quantitative monitoring of

student mastery through formative and summative evaluations, along with enhanced reading comprehension programs.

Medina's (2005) study at Negros Oriental State University found that students lacked necessary mathematical proficiency for College Physics I. The research revealed correlations between mathematical skills and factors such as gender, school type, and attitudes toward mathematics.

Further findings from Medina's study showed highest proficiency in right triangle solving and basic arithmetic, though mastery remained insufficient across all areas, with weakness in algebra and equation manipulation.

Dizon's (2008) research on Grade Four pupils in the Arayat East District showed satisfactory but improving performance in District Achievement Tests, attributing progress to school-implemented measures and regular testing.

Guzman's (2004) study of college freshmen at Pampanga Agricultural College identified specific challenging topics in College Algebra, particularly rational expressions, exponents, radicals, and degree equations. The research recommended increased instruction time for difficult topics and enhanced teacher development programs.

A study by Reston (2012) explored elementary mathematics teachers' probability concepts through inductive teaching methods. The research emphasized the effectiveness of inquiry-based and problem-based learning approaches in developing teachers' conceptual understanding. Reston's study concluded with recommendations for impact evaluation studies on teaching methods used in professional development programs, aiming to guide educational decision-makers in improving mathematics teacher preparation at the school level.

Conceptual Framework

The foundational premise of this study is that implementing a concrete-visual representation-abstract sequence approach will enhance learners' conceptual understanding and self-efficacy. Mathematics self-efficacy refers to an individual's unique abilities to perform organized and detailed work, while conceptual understanding involves grasping why mathematical ideas are integrated and functional rather than merely isolated facts (Pastornno & Doyle-Portillo, 2013).

Self-efficacy beliefs play a crucial role in decision-making processes (Hackett & Betz, 1981; May & Glynn, 2008). Margolis and McCabe (2006) define self-efficacy as one's belief in their capacity to achieve goals, noting that self-efficacious students can tackle difficult tasks with intrinsic motivation through mastery experiences, verbal persuasion, and emotional state.

Research by Fenci and Scheel (2005) emphasizes the importance of diverse teaching strategies in developing students' self-efficacy. The learning environment and teaching methods significantly influence self-efficacy development in the classroom (Bandura, 1991), with collaborative learning and inquiry-based activities showing particular promise in enhancing student self-efficacy.

Additional pedagogical approaches for improving self-efficacy include setting specific teaching goals, maintaining journals and reflection papers through portfolios, and evaluating student performance against set objectives rather than peer comparison (Schunk & Pajares, 2002; Bandura 1991).

Bandura (1977) identifies mastery experiences as the most significant source of efficacy. Research consistently shows that students with higher self-efficacy demonstrate greater learning motivation and persistence in facing challenges (Pajares & Graham, 1999; Pajares & Kranzler, 1995; Zeldin, Britner & Pajares, 2008).

The CVA approach aims to develop students' tangible understanding of mathematical concepts and skills (Special Connections, 2005). Misquitta's (2011) research demonstrated the effectiveness of combining CPA and explicit teaching approaches in improving students' performance in fraction equivalence tasks.

Hughes (2011) found that CPA sequenced instruction was particularly effective for struggling learners' acquisition, retention, and self-efficacy regarding fractions. The study revealed better retention of fraction knowledge compared to traditional instruction methods, even four weeks after completion. Research by Yagci (2010) on eighth-grade students showed statistically significant improvements in probability achievement through instruction with concrete models. Qualitative findings indicated positive student attitudes toward this instructional approach.

Witzel, Mercer, & Miller (2003) addressed students' struggles with algebra's abstract nature, emphasizing the value of concrete and pictorial representations in promoting understanding. Their study of 231 students demonstrated that those learning through the CPA model achieved significantly higher scores across all achievement levels. Flores (2010) noted that while the CPA instructional sequence has proven effective for basic mathematics computation, it extends to more complex mathematical processes. The research challenged the assumption that middle school students no longer need concrete representations. The study aims to evaluate the effectiveness of Concrete-Visual Representation-Abstract Sequence Instruction in teaching correlation and regression analyses to selected grade 11 students at Telesforo and Natividad Alfonso High School, using a descriptive mapping approach.

The initial phase focuses on assessing mathematical self-efficacy levels and conceptual understanding of Pearson product moment correlation coefficient and regression analysis in both experimental and control groups, before and after implementing the concrete-visual-abstract sequence approach. The second phase examines significant differences between pre-test and post-test results for both groups' conceptual understanding and mathematical self-efficacy levels when using the concrete-visual-abstract sequence approach. Further analysis investigates the relationship between mathematical self-efficacy and conceptual understanding levels in both experimental and control groups, before and after implementing the concrete-visual-abstract sequence approach.

The study concludes by evaluating the perceived benefits of the concrete-visual-abstract sequence approach on students' conceptual understanding of Pearson product moment correlation coefficient and regression analysis, considering various influencing factors through statistical analysis.

METHODOLOGY

This study employed a descriptive-experimental research design to evaluate the effectiveness of different teaching techniques in mathematics. The research utilized both pre-test and post-test measurements for experimental and control groups, combining descriptive analysis of mathematical self-efficacy and perceived benefits with experimental methodology.

The study involved Grade 11 students from Telesforo Natividad Alfonso High School in Sta. Maria, Sta. Ana, Pampanga during the 2021-2022 school year. Through lottery sampling, GAS 11-A (35 students) served as the experimental group, while GAS 11-B (38 students) functioned as the control group. The experimental group consisted of 62.86% males and 37.14% females, while the control group comprised 55.26% males and 44.74% females.

The research implemented two distinct strategies: the concrete-visual representational-abstract sequence approach for the experimental group and its inverse (abstract-visual representation-concrete) for the control group. The concrete approach utilized tangible materials for experiential learning, while visual representation employed videos and pictures to demonstrate Pearson product moment correlation and regression analysis. The abstract approach followed traditional chalk-and-talk methodology.

Data collection instruments included a 50-item multiple-choice test covering correlation and regression analysis, a 14-item mathematical self-efficacy questionnaire using a 7-point Likert scale, and a perceived benefits questionnaire using a 4-point Likert scale. The research process spanned two to three weeks, beginning with pre-tests and self-efficacy assessments, followed by the implementation of teaching strategies, and concluding with post-tests and final evaluations.

RESULTS AND DISCUSSION

This part presents the results, analysis, and interpretation of data gathered. The results were presented, analyzed, and interpreted to respond to the objectives and hypothesis of the study.

Level of Mathematical Self-Efficacy of the Experimental and Control Groups Before and After the Implementation of the Two Approaches

As to the level of mathematical self-efficacy of the experimental group before the implementation of the concrete - visual representation - abstract approach the three highest items are: item 3 - I believe I can complete all of the assignments in a statistics and probability subject (*6.1714, **1.5046, ***Quite True of Me); item 1 - I feel confident enough to ask questions in my statistics and probability class (*6.0286, **0.9231, ***Quite True of Me); and item 7 - I believe I am the type of person who can do statistics and probability (*5.9429, **1.0274, ***Quite True of Me). The grand mean of the experimental group in terms of the level of mathematical self-efficacy before the implementation of the

approach was 5.4245 with a standard deviation of 0.5286 and interpreted as *somewhat true of me*.

While, in terms of the level of mathematical self-efficacy of the control group before the implementation of the abstract - visual representation - concrete approach the three highest items are: item 1 - I feel confident enough to ask questions in my statistics and probability class (*6.3158, **1.0425, ***Quite True of Me); item 3 - I believe I can complete all of the assignments in a statistics and probability subject (*6.4474, **1.1786, ***Quite True of Me); and items 2 and 7 which have the same means - I believe I can do well on a statistics and probability test (*6.2632, **0.9777, ***Quite True of Me) and I believe I am the type of person who can do statistics and probability (*6.2632, **1.1551, ***Quite True of Me). The grand mean of the control group in terms of the level of mathematical self-efficacy before the implementation of the approach was 5.3421 with a standard deviation of 0.6972 and interpreted as *somewhat true of me*.

In terms of the level of mathematical self-efficacy of the experimental group after the implementation of the concrete - visual representation - abstract approach the three highest items are: item 8 - I believe I can learn well in a statistics and probability subject (*6.0571, **1.0556, ***Quite True of Me); item 9 - I feel confident when taking a statistics and probability test (*6.0857, **1.0396, ***Quite True of Me); and item 11 - I feel that I will be able to do well in future statistics and probability subject (*6.1143, **1.0784, ***Quite True of Me). The grand mean of the experimental group in terms of the level of mathematical self-efficacy after the implementation of the approach was 5.8327 with a standard deviation of 0.4161 and interpreted as *quite true of me*.

Finally, in terms of the level of mathematical self-efficacy of the control group after the implementation of the abstract - visual representation - concrete approach the three highest items are: item 1 - I feel confident enough to ask questions in my statistics and probability class (*6.7895, **0.4741, ***Very True of Me); item 3 - I believe I can complete all of the assignments in a statistics and probability subject (*6.8684, **0.3426, ***Very True of Me); and item 7 - I believe I am the type of person who can do statistics and probability (*6.6053, **0.7548, ***Very True of Me). The grand mean of the control group in terms of the level of mathematical self-efficacy after the implementation of the approach was 6.4436 with a standard deviation of 0.4496 and interpreted as *quite true of me*.

As a conclusion, before the implementation of the two approaches to the two different groups, both experimental and control groups got the same items of 1 (I feel confident enough to ask questions in my statistics and probability class) and 7 (I believe I am the type of person who can do statistics and probability) as their highest items and item 5 (I believe I will be able to use statistics and probability in my future career when needed) as their lowest similar item.

After the implementation of the two approaches, experimental and control groups changed their options, items 1 and 7 remain as the highest items of the control group, while the experimental group's items changed from 1, 3,

and 7 to 8, 9, and 11. The same thing occurs for the lowest items as the control group's item 13 (I believe I can think like a mathematician in statistics and probability) remains while item 5 (I believe I will be able to use statistics and probability in my future career when needed) remains as the lowest item for the experimental group. Margolis and McCabe (2006) define self-efficacy as the conviction that one can accomplish his own goals. Students that have self-efficacy are capable of overcoming challenges and possess an innate drive. Additionally, they highlighted how verbal persuasion, emotional states, and mastery experiences help students develop self-efficacy.

Level of Conceptual Understanding of the Experimental and Control Groups

Based on the result of the first implementation of the test, the control group from this research got a mean score of 11.8286 which was Fair (Fa) with a standard deviation of 2.8022. The experimental group got a mean score of 11.2857 which was Fair (Fa) with a standard deviation of 2.5961. This means that both groups had a little idea in terms of Pearson product moment correlation and regression analysis. At the same time, it shows clearly that the control group had greater ideas in the said topics using the least significant difference of 0.5429.

On the other hand, based on the result of the second implementation of the test, the control group from this research got a mean score of 38.0000 which was Satisfactory (S) with a standard deviation of 7.3804. The experimental group got a mean score of 27.6000 which was Good (G) with a standard deviation of 7.0594. This means that both groups had learned Pearson product moment correlation and regression analysis from the two given strategies (concrete -visual representation - abstract approach and abstract - visual representation - concrete approach). At the same time, it shows clearly that the control group had greater learning in the said topics using the least significant difference of 10.4000.

Additionally, it was shown that the most learned and least learned items for the experimental group after the implementation of concrete-visual representation - abstract sequence of instructional approach. It was shown in the result that most of the experimental group's most learned competencies were Pearson product moments of correlation, and their least learned competencies were regression analysis. It was also found in the study that most of the experimental group found difficulties in solving regression analysis. On the other hand, most of this group is good in understanding concepts about the topics. While the most learned and least learned items for the control group after the implementation of abstract-visual representation - concrete sequence of instructional approach. It was shown in the result that most of the experimental group's most learned competencies was on regression analysis and their least learned competencies was Pearson product moment correlation. It was also found in the study that most of the control group found difficulties in solving Pearson r . On the other hand, most of this group is good in understanding concepts about the topics.

Difference between the Pretest and Posttest Results of the Control and Experimental Groups

Using t-test for paired sample statistics, the study obtained -11.793 for the pre-test and post-test difference of the experimental group with a p-value of 0.000 and interpreted as significant. The study obtained -22.847 for the pre-test and post-test difference of the control group with a p-value of 0.000 which was interpreted as significant. This concludes that there is a significant difference between the pre-test and post-test results of the control and experimental groups. This simply means that both concrete –visual representation – abstract approach and abstract – visual representation – concrete approach was effective in teaching Pearson product moment correlation and regression analysis.

In addition, before the implementation of the concrete – visual representation – abstract approach for the experimental group, respondents obtained a mean of 5.4245 level of mathematical self-efficacy, which was interpreted as somewhat true of me (ST) with standard deviation of 0.5286. After the implementation of the said approach, the experimental group obtained an average of 5.8327 levels of mathematical self-efficacy which was interpreted as quite true of me (QT) with a standard deviation of 0.4161. Using t-test for paired sample statistics, the study obtained -6.268 with a p-value of 0.000 and interpreted as significant.

Before the implementation of the abstract – visual representation – concrete approach for the control group, respondents obtained a mean of 5.3421 level of mathematical self-efficacy, which was interpreted as somewhat true of me (ST) with standard deviation of 0.6975. After the implementation of the said approach, the control group obtained an average of 6.4436 levels of mathematical self-efficacy which was interpreted as very true of me (VT) with a standard deviation of 0.4494. Using t-test for paired sample statistics, the study obtained -8.900 with a p-value of 0.000 and interpreted as significant.

This concludes that there is a significant difference between the level of mathematical self-efficacy of the control and experimental groups before and after the implementation of the two approaches. This simply means that both concrete –visual representation – abstract approach and abstract – visual representation – concrete approach was effective in teaching Pearson product moment correlation and regression analysis. Individuals' views or opinions about their mathematical talents, as well as how they prefer to learn and how quickly they learn, are generally referred to as mathematics self-efficacy. Students' mathematical performance was impacted by their level of mathematics self-efficacy, according to Higbee and Thomas (1999). According to their study's findings, some pupils cannot learn mathematics by merely concentrating on teaching mathematical knowledge. As a result, the topic should be covered gradually so that students can comprehend the lessons through conversation, modeling, or representations.

Difference between the Experimental and Control Groups Level of Conceptual Understanding before and after the Implementation of the Two Approaches

The difference between the pre-test results of the control and experimental groups shows using T-test for independent samples as an

indicator the result was 0.840 with a p-value of 0.407 and interpreted as not significant. This means that there was no significant difference between the results of the pretest of the control and experimental groups in Pearson product moment correlation and Regression analysis. Also, the difference between the post-test results of the control and experimental groups using the same indicator obtained a value between the two variables was 5.941 with a p-value of 0.000 which was respectively significant.

This means that there was a significant difference between the post-test results of the control group and the experimental group. From the result of the mean score for the experimental group during the pre-test which was 11.2857 and interpreted as fair (Fa) it was turned into 27.6000 mean score and interpreted as good (S) with the increase on the level of assessment which is equal to 32.63%. From the result of the mean score for the control group during the pre-test which was 11.8286 and interpreted as fair (Fa) it was turned into 38.0000 mean score and interpreted as good (S) with the increase on the level of assessment which is equal to 52.34 %, significantly showing that the abstract - visual representation - concrete approach used for the control group in teaching Pearson product moment of correlation and regression analysis was significantly effective as compare to the concrete - visual representation - abstract approach used for the experimental group.

The abstract-visual representation-concrete instructional approach is one of the most successful methods in mathematics because it shows how students gradually grasp and learn the subject through behaviorist modeling that culminates in exploratory concrete experiences. This gives the students a fantastic opportunity to acquire the content under the teacher's careful supervision before working independently. The study of Canonizado (2009), who claims that children learn and appreciate mathematics through the experiences that teachers give them, lends credence to this hypothesis. Nonetheless, Scarpello (2007) also noted the beneficial impact a teacher can have on a pupil.

Difference between the Experimental and Control Groups' Level of Mathematical Self-Efficacy before and after the Implementation of the Two Approaches

Using T-test for independent samples as an indicator the result was - 0.688 with a p-value of 0.496 and interpreted as not significant. This means that there was no significant difference between the experimental and control groups' level of mathematical self-efficacy before the implementation of the two approaches. Both groups have the same level of mathematical self-efficacy. Also, it shows the difference between the post-test results of the control and experimental groups using the same indicator the value obtained between the two variables was 4.964 with a p-value of 0.000 which was respectively significant. This means that there was a significant difference between the level of mathematical self-efficacy of the control and experimental groups after the implementation of the two approaches. This shows that the control group improved more self-efficacy level than the experimental group using the

abstract - visual representation - concrete approach in teaching Pearson product moment correlation and regression analysis.

The relationship between math performance and math self-efficacy is favorable (Pajares & Miller, 1994; Kabiri & Kiamanesh, 2004; Liu & Koirala, 2009). This implies that an individual's performance in solving mathematical issues increases with their score on mathematics self-efficacy measures. According to Odaci (2011), "students' belief in their academic self-efficacy and their ability to begin and continue their studies is also highly important" (p. 110). High academic self-efficacy students are more self-assured and have more optimistic outlooks on their future careers.

Perceived Benefits of the Experimental and Control Groups on the Two Approaches

The level of perceived benefits of the experimental group after the implementation of the concrete - visual representation - abstract approach was revealed. The leading highest items are: item 3 - I could easily connect its concepts to my previous experiences (*3.5143, **0.6585, ***Strongly Agree); item 7 - I felt fulfilled when I got a passing score in statistics and probability class (*3.5714, **0.5576, ***Strongly Agree); item 10 - I could participate with different statistics and probability activities very easily especially with my classmates (*3.4857, **0.5621, ***Agree); and item 11 - I felt fulfilled in understanding the content of the lessons (*3.4857, **0.5621, ***Agree). The grand mean of the experimental group in terms of the level of perceived benefits after the implementation of the approach was 3.3657 with a standard deviation of 0.2103 and interpreted as *Agree*. This CRA approach benefits all students but has been shown to be particularly effective with students who have mathematics difficulties, mainly because it moves gradually from actual objects through pictures and then to symbols (Jordan, Miller, & Mercer, 1998).

It was also revealed through this study the level of perceived benefits of the control group after the implementation of the abstract - visual representation - concrete approach. The leading highest items are: item 6 - I was motivated to analyze and find the correct solution when I commit error or mistakes (*3.5789, **0.5004, ***Strongly Agree); item 11 - I felt fulfilled in understanding the content of the lessons (*3.5789, **0.5004, ***Strongly Agree); item 13 - The statistics and probability lessons became more exciting even if challenging (*3.6053, **0.4954, ***Strongly Agree); and item 14 - I became more willing to participate in learning statistics and probability lessons (*3.5789, **0.5004, ***Strongly Agree). The grand mean of the control group in terms of the level of perceived benefits after the implementation of the approach was 3.4842 with a standard deviation of 0.1311 and interpreted as *Agree*.

This means that abstract - visual - representation - abstract instructional approach is a facilitating framework for students to create meaningful connections between theories, representations, and concrete levels of thinking and understanding. Student's learning starts out with an abstract level of thinking, where students are exclusively using mathematical symbols to represent and model problems and then students can extend their knowledge

through visual representations and then finally are able to move to the visual, tangible, and kinesthetic experiences to establish deeper understanding.

CONCLUSION AND RECOMMENDATIONS

This study concludes that mathematical self-efficacy is positively related to conceptual understanding. Students with high level academic self- efficacy are more self-confident and have more positive attitudes towards mathematics. Abstract – Visual Representation - Concrete instructional approach appears to be an effective approach in teaching Pearson product moment correlation and regression analysis that provides a clear picture of how students understand and learn those lessons gradually through modelling which is behaviorist in nature and ended through concrete experiences which is exploratory in nature. This offers a great opportunity for the learners to learn the material first through the proper guidance of the teacher before they do it by themselves. Student's learning starts out with an abstract level of thinking, where students are exclusively using mathematical symbols to represent and model problems and then students can extend their knowledge through visual representations and then finally are able to move to the visual, tangible, and kinesthetic experiences to establish deeper understanding. This theory is supported by the study of Canonizado (2009) that, according to him, students learn and enjoy Mathematics through the experiences that teachers provide. Also, Scarpello (2007), mentioned the positive influence an instructor can have on a student.

ADVANCED RESEARCH

Several limitations should be considered when interpreting the results of this study. The research was conducted in a single school (Telesforo Natividad Alfonso High School) with a relatively small sample size of 73 students (35 in the experimental group and 38 in the control group) from the General Academic Strand. This limited scope may affect the generalizability of the findings to other educational contexts, student populations, or academic programs. Additionally, the study's duration of two to three weeks may not have been sufficient to fully assess the long-term impact of the Abstract-Visual-Concrete approach on students' mathematical self-efficacy and conceptual understanding.

The study also faced methodological limitations in controlling external variables that might have influenced student performance and self-efficacy. Factors such as students' prior mathematical experiences, socioeconomic background, access to additional learning resources, and individual learning styles were not extensively controlled or analyzed. Furthermore, the research relied heavily on quantitative measures through pre-tests, post-tests, and Likert-scale questionnaires, potentially missing valuable qualitative insights into students' learning experiences and cognitive processes during the implementation of the teaching approaches.

For future research, several avenues of investigation are recommended. Researchers should consider conducting longitudinal studies to examine the sustained effects of the Abstract-Visual-Concrete approach on mathematical achievement and self-efficacy beyond the immediate post-intervention period.

Additionally, comparative studies across different school types, grade levels, and socioeconomic contexts would provide more comprehensive insights into the approach's effectiveness. Future studies could also incorporate mixed-method designs, including classroom observations, student interviews, and focus group discussions, to gain deeper understanding of how students engage with and internalize mathematical concepts through this instructional approach. Investigation into the role of technology integration within the Abstract-Visual-Concrete framework could also yield valuable insights for modern educational practices.

REFERENCES

- Batanero, C. & Diaz, C. (2011). Training school teachers to teach probability: Reflections and challenges. *Chilean Journal of Statistics*. Retrieved December 2014 from: http://chjs.deuv.cl/iFirst_art/ChJS010202.pdf
- Bennie, K., Hudson, K., & Newstead, K. (1999). Statistics Probability 2 Teachers Document. Mathematics Learning and Teaching Initiative, 1-26. Retrieved last January 9, 2015 from <http://academic.sun.ac.za/mathed/MALATI/2Prob2.pdf>
- Bruner, J. S. (1960). "The growth of representational processes in childhood." In J. M. Anglin (Ed.), *Beyond the information given: Studies in the psychology of knowing* (pp. 313-324). New York, NY: Norton.
- Carr, A., Jonassen H., Lit Zinger E., Marra , M. (1998) Good news to foment educational revolution: the role of systematic change in advancing situated learning, constructivism, and feminist pedagogy. *Journal of Educational Technology*, 38(1):5-15
- David, A.G. (2007). The Mathematics Performance Of PAC College Freshmen as Influence by Different Factors. (Masters' Thesis, Pampanga Agricultural College, Magalang, Pampanga.)
- Definition of CRA: Retrieved October 15, 2014, from <http://www.broward.k12.fl.us/studentssupport/ese/PDF/CRAApproach inMath.pdf>
- Definition of K to 12: Retrieved October 15, 2014, from <http://www.deped.gov.ph/k-to-12/About/features>
- Definition of Spiral Curriculum: Retrieved October 15, 2014, from <http://everydaymath.uchicago.edu/about/why-it-works/spiral/>
- Dizon, J. C., (2008). Performance of Grade Four Pupils of Arayat East District in the Division Achievement Test. (Masters' Thesis, Pampanga Agricultural College, Magalang, Pampanga.)

- Encyclopedia Britannica. Definition of Mathematics. Retrieved January 29, 2015, from www.en.wikipedia.org
- Flores, M. M. (2010). "Using the concrete-representational-abstract sequence to teach subtraction with regrouping to students at risk for failure". *Remedial and Special Education, 31*(3), 195-207. doi: 10.1177/0741932508327467
- Garfield, J., & Ahlgren, A. (1988). Difficulties in learning basic concepts in probability and statistics: Implications for research. *Journal for Research in Mathematics Education, 19*, 44-63.
- Gomez, B. G. (2010). Predictors of Mathematics Ability of Fourth Year High School Students in Mabalacat, Pampanga. (Masters' Thesis, Pampanga Agricultural College, Magalang, Pampanga.)
- Guzman, E. S. (2004). Difficulties Encountered by Freshman Students in College Algebra at Pampanga Agricultural College Magalang. (Masters' Thesis, Pampanga Agricultural College, Magalang, Pampanga.)
- Hudson, P., Miller, S. P., & Butler, F. (2006). Adapting and merging explicit instruction within reform based mathematics classrooms. *American Secondary Education, 35*(1), 19.
- Hughes, E. M. (2011). "The Effects of the Concrete-Representational-Abstract Sequenced Instruction on Struggling Learners Acquisition, Retention and Self-efficacy of Fraction". (Dissertation) Graduate School of Clemson University
- Introduction to probability, Retrieved last January 10, 2015 from <https://www.youtube.com/watch?v=-8eSOmTPUbk>
- Johnson D. M. (1989). "Probability and Statistics". South-Western Publishing Co. Cincinnati, Ohio, (250-357)
- Jordan, L., Miller, M., & Mercer, C. D. (1998). The effects of concrete to semi-concrete to abstract instruction in the acquisition and retention of fraction concepts and skills. *Learning Disabilities: A Multidisciplinary Journal, 9*: 115-122.
- Kaput, J. (1992). "Technology and mathematics education." In D.A. Grouws (Ed.) *Handbook of research on mathematics and teaching and learning* (515-556), New York: Macmillan Publishing Company.
- Keeler, C. & Steinhorst, K. (2001). A new approach to learning probability in the First Statistics Course. *Journal of Statistics Education, 9*(3). randomness

and probability: An empirical study. *Statistics Education Research Journal*,

- Luke, J. (2012). "The Impact of Manipulatives on Students' Performance on Money Word Problems". Georgia State University
- Mahanta, D. (2012). "Achievement in Mathematics: Effect of Gender and Positive/Negative Attitude of Students". Department of Mathematics, Nowgong Girls' College, Nagaon, (Assam)
- Mathematics Learner's Module 8. Department of Education, third Floor, Bonifacio Building, DepEd Complex, Meralco Avenue, Pasig City, Philippines 1600, 315 - 481
- Medina, R. P. (2005). The Mathematics Readiness of Students in Taking General College Physics I at Negros Oriental State University. Retrieved February 25, 2015, from <http://physics.msiit.edu.ph/spum/papers/>
- Misquitta, R. (2011). "Teaching Fractions to Middle-School Students Struggling in Mathematics: An Exploratory Study". (Doctoral Dissertation), University of Texas, USA
- Moore, D. S. (1997). Probability and Statistics in the Core Curriculum. *Confronting the Core Curriculum*, 93-98. Retrieved February 19, 2015 from <http://www.stat.purdue.edu/~dsmoore/articles/StatInCore.pdf>
- National Council of Teachers of Mathematics, Commission on Standards for School Mathematics. (1989). "Curriculum and Evaluation Standards for School Mathematics. Reston", VA: Author
- Pangan, G. L. (2010). Exploring the Use of the Concrete-Representational-Abstract Approach in Teaching Probability. Masters' Thesis, Bulacan State University, Bulacan.
- Papaieronymou, I. (2009). Recommended Knowledge of Probability for Secondary Mathematics Teachers. Proceedings from Proceedings of CERME 6, Lyon, France.
- Parreno, E. and Jimenez R. (2006). Basic Statistics Worksheet. C & E Publishing, Inc. 839 EDSA, South Triangle, Quezon City, Philippines, 195 - 216.
- Pratt, D. (2011). Re-connecting probability and reasoning about data in secondary school teaching. Proceedings from 58th World Statistics Congress of the International Statistical Institute.

- Quismundo, T. (2011). 2/3 of the Philippine high school obtain low NAT scores. Retrieved on September 5, 2014, from <http://newsinfo.inquirer.net/63039/23-of-philippines-high-schools-obtain-low-nat-scores>.
- Reston, E. & Bersales, L. G. (2011). Reform Efforts in Training Mathematics Teachers to Teach Statistics: Challenges and Prospects. In C. Batanero, G. Burril & C. Reading (Eds.) *Teaching Statistics in School Mathematics - Challenges for Teaching and Teacher Education: A Joint Study ICMI/IwASE Study Book*. Heidelberg: Springer.
- Reyes J.D. Mathematics Anxiety and Self-Efficacy: A Phenomenological Dimension. *Journal of Humanities and Education Development (JHED)*, 2019. Vol. 1, no. 1, pp. 22-34. URL: <https://mail.theshillonga.com/index.php/jhed/article/view/9/4>
- Reyes, J. D. C. (2019). Team-Pair-Solo: An Experimental Approach in Teaching Random Variables and Discrete Probability Distributions. *Journal of Humanities and Education Development (JHED)*, 1(1), 35-45. Retrieved from <http://www.theshillonga.com/index.php/jhed/article/view/8>
- Reyes, J. D., (2019). Increasing Self-efficacy and Alleviating Anxiety using Touch Math and Instructional Games: An Intervention for Low Performing Seventh Graders. *Journal of Humanities and Education Development (JHED)*. ISSN: 2581-8651. Vol-1, Issue-2, Mar - Apr 2019. Retrieved from <https://theshillonga.com/index.php/jhed/article/view/13>
- Reyes, J. D., (2020). Effectiveness of Concrete-Representational-Abstract sequence of instruction in probability on select-groups of eight graders. *International Journal of Multidisciplinary Research and Growth Evaluation*. Volume 2; Issue 1; January-February; 2021; Page No. 438-443.
- Reyes, J. D., Insorio, A. O., Ingreso, M. V., Holario, F. F., & Gutierrez, C. R., (2019). Conception and Application of Contextualization in Mathematics Education. *International Journal of Educational Studies in Mathematics*, 2019, 6(1), 1-18. Retrieved from <https://dergipark.org.tr/en/download/article-file/664590>
- Ignacio Jr, A. G., & Reyes, J. D. (2017). Exploring Mathematics Achievement Goals Using Kolb's Learning Style Model. *Asia Pacific Journal of Multidisciplinary Research*, 5(1), 19-24.
- Shaughnessy, M. (1977). Misconceptions of Probability: An experiment with a small-group, activity-based, model building approach to introductory

probability at the college level. *Educational Studies in Mathematics*, 8, 295-316.

Special Connections, (2005). From concrete to representational to abstract. Retrieved January 9, 2015, from the Special Connections Web site: <http://www.specialconnections.ku.edu/cgi-bin/cgiwrap/specconn/main.php?cat=instruction&subsection=math/cra>.

Throwing a Die/ Dice, Retrieved last January 10, 2015 from <https://www.youtube.com/watch?v=8ZsEYKN0ht8>

Tossing a Coin, Retrieve last January 10, 2015 from <https://www.youtube.com/watch?v=5lpqiGixDd0> and <https://www.youtube.com/watch?v=ExUmsArDXUM>

Watt, S.J. (2013). "Teaching algebra-based concepts to students with learning disabilities: The effects of pre teaching using a gradual instructional sequence". University of Iowa

Witzel, B. S. (2005). "Using CRA to Teach Algebra to Students with Math Difficulties in Inclusive Settings Learning Disabilities": *A Contemporary Journal* 3(2), 49-60

Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). "Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model". *Learning Disabilities Research and Practice*, 18, 121-131

Yagci, F. (2010). "The Effect of Instruction with Concrete Models on Eight Grade Students' Probability Achievement and Attitudes toward Probability". (Thesis), Middle East Technical University, Turkey