



Optimization Policies for Inventory Model with Deterministic Demand Based on Selling Price and Promotional Efforts Under Inflationary Conditions

Rupali Jindal^{1*}, Arun Krushna Padhihari², Leena Prasher³

^{1,2}Research Scholar, Department of Mathematics, CT University, Ludhiana, India

³Department of Mathematics, CT University, Ludhiana, India

Corresponding Author: Rupali Jindala j.rupali100@gmail.com

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ABSTRACT

This paper constructs an economic order quantity model considering demand dependent upon selling price, time and promotional efforts made by the firm/sales team. Items are deteriorating with a constant rate. The cost for holding the items is taken as constant. Cost of advertisement is assumed to be quadratic function of marketing efforts. The objective of the paper is to determine economic replenishment quantity in inflationary conditions so as to maximize profit. Numerical example is presented to illustrate the theory. Graphical presentation is made to describe the optimality of the model with suitable sensitivity analysis

INTRODUCTION

The main concern of a business unit is to decide ordering lot size keeping in view various constraints associated with the demand rate, deterioration rate, inflation, holding costs, ordering costs, advertisement costs, etc. The nature of varied situations in different combinations affect the management standpoint on ordering lot size measure. The accurate ordering time seeking maximum profit and customer satisfaction is the basic objective of any organization. This paper deals with above said issues in situation when demand is changing according to changing time and selling price of the product in addition to promotional effects of marketing tactics such as marketing of new technology launched, timely customer service, increased warranty period, etc. In traditional models, researchers used to take constant demand. But in real market situation, this assumption is hypothetical. Demand may increase or decrease with time depending on various other factors. Bose et al (1995) developed an inventory model for deteriorating items

with linear time-dependent demand rate, shortages under inflation and time discounting. Deterministic model of perishable inventory with stock dependent demand and non-linear holding cost was developed by Giri and Chaudhury (1998). They were of the opinion that increase in the holding cost increases total inventory cost. Moon et al. (2005) has taken the deterioration with additional issue of ameliorating while developing inventory replenishment policies considering inflation and time value of money as important parameters. Singh and Singh (2011) developed a model for imperfect production system deteriorating under inflationary conditions with exponential demand. Mishra et al. (2012) in his paper discussed the inventory system for perishable goods and has taken the demand as a function of inflation. Inventory level of the supplier while there is deterioration in the system is analyzed by Ghiami and Williams (2015). Shah and Vaghela (2016) in their paper gave the idea of demand dependent upon time and advertisement under the effect of inflation. Chan et al. (2017) has discussed the deterioration of items during delivery. He tried to determine the production rate of exponentially deteriorating items with the aim of minimizing costs. Pando et al. (2018) in his paper has assumed the demand rate for deteriorating items dependent upon inventory level. He has also made an attempt to formulate a model for profit maximization in spite of minimizing costs. San-Lose et al. (2018) has analyzed profit maximization with demand as a function of price and time both. Advertisement and inflation are growing as important factors while planning for inventory. Bose et al. (1995), Ray and Chaudhuri (1997), Kun-Shan Wu (2001) and Ouyang et al. (2007) has discussed the EOQ model with the effects of time discounting and inflation. They discussed inflation rates in two different concepts:- One is company inflation rate and other is economy inflation rate. In today's modern world, advertisement plays an important role in increasing the sales of a product. There are number of available options before a customer. he has to decide which things to buy. Good quality of advertisement leaves a mark on your memory which helps you in your future purchasing. Singh and Jain

(2009) discussed an EOQ model on reserve money under inflation and suppliers credit. Effect of inflation is studied by Singh et al in 2010 on an inventory model for two shops under one management selling deteriorating items having stock dependent demand with shortages. Thangam and Uthayakumar (2010) also gave a approach of discounted cash flow for deteriorating items with exponential partial backorders and inflation induced demand.

DISCUSSION

Notations and Assumptions

Notations

A Purchase cost per item (currency units/order)

$q(t)$ on hand inventory at time t

$\gamma = r - i$ where r is the interest per unit currency and i is the inflation per unit currency

hc inventory holding cost per unit item per unit time (currency units)

$M(t)$ efforts made by sales team at time t

$H(t)$ cost of efforts $M(t)$ by the sales team

s selling price per unit (currency)

S initial inventory level (decision variable)

Assumptions

1. There is single item in the system.
2. Shortages are not allowed.
3. Lead time is zero.
4. There is constant rate of deterioration of items with no repair or replacement.
5. The replenishment at any time t is a scalar multiple of available inventory level i.e. $aq(t)$, ($a > 0$).
6. Demand rate $R(t)$ is given by $R(t) = (b_1 - b_2s + b_3t) + \lambda M(t)$ where $b_1 > 0, b_2 > 0, b_3 > 0, \lambda > 0; M(t) \geq 0$.
7. The cost of efforts $H(t) = c_1(M(t))^2 + c_2M(t) + c_3$ where $c_1 > 0, c_2 > 0, c_3 > 0$ whose values depend upon the business environment

Mathematical Modeling

In this model, the consumption of the inventory is due to deterioration and demand of the product which depends directly upon the selling price, time and marketing efforts. The initial stock for replenishment time interval $[0, T]$ is supposed to be S and constant deterioration rate is θ . The governing differential equation for the inventory level $q(t)$ at any time t is as below:

$$\frac{dq}{dt} + (\theta - a)q(t) = -(b_1 - b_2s + b_3t) - \lambda M(t); q(0) = S, q(T) = 0 \quad (1)$$

Rearranging above equation, we get

$$M(t) = \frac{1}{\lambda} [(a - \theta)q(t) - q' - (b_1 - b_2s + b_3t)] \quad (2)$$

The profit function for the replenishment time interval $[0, T]$ under the effect of inflation is as below:

$$\eta = \int_0^T e^{-\gamma t} [sR(t) - hcq - H(t) - A\theta q] dt$$

Using value of $M(t)$ from (2) in above equation, we get

$$\begin{aligned} \eta &= \int_0^T e^{-\gamma t} [s(b_1 - b_2s + b_3t) + s\lambda M(t) - hcq - A\theta q - \frac{c_1}{\lambda^2} ((a - \theta)q - q' - (b_1 - b_2s + b_3t))^2 \\ &\quad - \frac{c_2}{\lambda} ((a - \theta)q - q' - (b_1 - b_2s + b_3t)) - c] dt \\ &= \int_0^T e^{-\gamma t} [s(b_1 - b_2s + b_3t) + \frac{s\lambda - c_2}{\lambda} ((a - \theta)q - q' - (b_1 - b_2s + b_3t)) - (hc + A\theta)q \\ &\quad - \frac{c_1}{\lambda^2} ((a - \theta)q - q' - (b_1 - b_2s + b_3t))^2 - c] dt \end{aligned} \quad (3)$$

Equation (3) can also be written as

$$\eta = \int_0^T f(t, q, q') dt \text{ with } q(T) = 0 \text{ and } q(0) = S \text{ where}$$

$$\begin{aligned} f(t, q, q') &= e^{-\gamma t} [s(b_1 - b_2s + b_3t) + \frac{s\lambda - c_2}{\lambda} ((a - \theta)q - q' - (b_1 - b_2s + b_3t)) - (hc + A\theta)q \\ &\quad - \frac{c_1}{\lambda^2} ((a - \theta)q - q' - (b_1 - b_2s + b_3t))^2 - c] \end{aligned} \quad (4)$$

In this model, the purpose is to derive the optimal paths $q(t)$ and $M(t)$ which gives maximum profit η .

Lemma

If there exists a path $q(t)$ for the functional $\eta = \int_0^T f(t, q, q') dt$ such that $\frac{\partial f}{\partial q} - \frac{d}{dt} \left(\frac{\partial f}{\partial q'} \right) = 0$ and $\frac{d^2 \eta_\delta}{(d\delta)^2} \Big|_{\delta=0}$ has negative value then in $[0, T]$, η attains its maximum value.

Proof: Let us assume that on the path $q(t)$, η is dependent. Now, consider the family $q_\delta(t) = q(t) + \delta\psi(t)$, with $\psi(0) = \psi(T) = 0$, (so that $q_\delta(t)$ and $q(t)$ agree on common boundary conditions). Here δ is a parameter, so need not be small. Now, consider the functional $\eta_\delta = \int_0^T f(t, q_\delta, q'_\delta) dt$. Above integral is a function of δ for a given ψ . As we know, $q(t)$ is the function giving maximum value to η . Therefore, the integral η_δ attains maximum value at $\delta = 0$ (when $q_\delta = q$) and $\frac{d\eta_\delta}{d\delta} \Big|_{\delta=0} = 0$. Now, differentiating under the sign of integration

$$\begin{aligned} \frac{d\eta_\delta}{d\delta} &= \int_0^T \left[\frac{\partial f_\delta}{\partial \delta} + \left(\frac{\partial f_\delta}{\partial q_\delta} \frac{\partial q_\delta}{\partial \delta} + \frac{\partial f_\delta}{\partial q'_\delta} \frac{\partial q'_\delta}{\partial \delta} \right) \right] dt \\ &= \int_0^T \left[\frac{\partial f_\delta}{\partial \delta} + \left(\frac{\partial f_\delta}{\partial q_\delta} \psi(t) + \frac{\partial f_\delta}{\partial q'_\delta} \psi'(t) \right) \right] dt \end{aligned} \tag{5}$$

Using integration by parts on the second term in above integral and applying boundary conditions on ψ , we get

$$\frac{d\eta_\delta}{d\delta} = \int_0^T \psi(t) \left[\frac{\partial f_\delta}{\partial \delta} - \frac{d}{dt} \left(\frac{\partial f_\delta}{\partial q'_\delta} \right) \right] dt \tag{6}$$

Hence, $\left(\frac{d\eta_\delta}{d\delta} \right)_{\delta=0} = 0$ gives us

$$\frac{\partial f}{\partial q} - \frac{d}{dt} \left(\frac{\partial f}{\partial q'} \right) = 0$$

Above equation is the necessary condition to find the extremum of η known as the Euler-Lagrange equation. Now

$$\frac{d^2 \eta_\delta}{(d\delta)^2} = \int_0^T \left(\psi^2 \frac{\partial^2 f_\delta}{\partial q_\delta^2} + 2\psi\psi' \frac{\partial^2 f_\delta}{\partial q_\delta \partial q'_\delta} + \frac{\partial^2 f_\delta}{\partial q'^2} \right) dt \tag{7}$$

which implies

$$\left(\frac{d^2 \eta_\delta}{(d\delta)^2} \right)_{\delta=0} = \int_0^T \left(\psi^2 \frac{\partial^2 f}{\partial q^2} + 2\psi\psi' \frac{\partial^2 f}{\partial q \partial q'} + \frac{\partial^2 f}{\partial q'^2} \right) dt \tag{8}$$

Evaluating each derivative required in (9) and substituting values, we get

$$\frac{d^2\eta}{d\delta^2} = - \frac{\int_0^T \frac{2c_1}{\lambda^2} e^{-\gamma t} [\psi(a-\theta) - \psi]^2 dt}{\int_0^T e^{-\gamma t} [\psi(a-\theta) - \psi]^2 dt} < 0, \text{ as } \int_0^T e^{-\gamma t} [\psi(a-\theta) - \psi]^2 dt > 0 \quad (9)$$

It is, therefore, proved that η has maximum value as the sufficient condition for maxima is satisfied.

Applying the condition for extremum and using value of $f(t, q, q')$ from (4), we get

$$q'' - \gamma q' - (a-\theta)(a-\theta-\gamma)q = - \frac{\lambda^2}{2c_1} [s(a-\theta-\gamma) - (hc + A\theta)] + \frac{\lambda c_2}{2c_1} (a-\theta-\gamma) - (a-\theta-\gamma)(b_1 - b_2s + b_3t) - b_3 \quad (10)$$

Solving above differential equation, we get

$$\frac{q(t)}{\theta^3 t} = \frac{K_1 e^{-(a-\theta-\gamma)t} + K_2 e^{(a-\theta)t} + \frac{\lambda^2 [s(a-\theta-\gamma) - (hc + A\theta)] + \frac{\lambda c_2}{2c_1} (a-\theta-\gamma) - (a-\theta-\gamma)(b_1 - b_2s + b_3t) - b_3}{(a-\theta)^2} + K_3}{(a-\theta)} \quad (11)$$

where k_1 and K_2 are arbitrary constants to be determined and k_3 is given by

$$K_3 = \frac{\lambda^2 [s(a-\theta-\gamma) - (hc + A\theta)] + \frac{\lambda c_2}{2c_1} (a-\theta-\gamma) - (a-\theta-\gamma)(b_1 - b_2s + b_3t) - b_3}{(a-\theta)(a-\theta-\gamma)} - \frac{\lambda c_2}{2c_1} (a-\theta) + a - \theta \quad (12)$$

Here, $a > (\theta + \gamma)$, otherwise $I \rightarrow \infty$. Also $t \rightarrow \infty$ is not suitable for marketing system. Applying boundary conditions, we get

$$\frac{K_2 e^{-(a-\theta)T} + K_2(1 - \frac{b_3 T}{e^{(a-\theta)T} - e^{-(a-\theta-\gamma)T}})}{e^{(a-\theta)T} - e^{-(a-\theta-\gamma)T}} \quad (13)$$

and

$$\frac{K_2(e^{-(a-\theta-\gamma)T} - 1)}{e^{-(a-\theta-\gamma)T} - e^{(a-\theta)T}} \frac{b_3 T}{(a-\theta)(e^{-(a-\theta-\gamma)T} - e^{(a-\theta)T})}$$

Hence, the optimal paths $M(t)$ and $q(t)$ are given by

$$M(t) = \frac{1}{\lambda} [(2(a-\theta) - \gamma)K_1 e^{-(a-\theta-\gamma)t} + K_2(a-\theta) - \frac{b_3}{a-\theta} - b_2 s] \quad (15)$$

and

$$q(t) = \frac{K_1 e^{-(a-\theta-\gamma)t} + K_2 e^{(a-\theta)t} + \frac{b_3}{(a-\theta)} + K_3}{\lambda} \quad (16)$$

Substituting optimal values of $q(t)$ and $M(t)$ in (4) and simplifying, we get

$$\eta(S) = (K_1(S))^2 Y_1 + K_1(S) Y_2 + K_2(S) Y_3 + Y_4 \quad (17)$$

where

$$Y_1 = \frac{c_1}{\lambda^2} (2(a-\theta) - \gamma) [e^{-(2(a-\theta-\gamma)T} - 1],$$

$$Y_2 = \frac{1}{\lambda^2} [(s + \frac{c_2}{\lambda}) (2(a-\theta) - \gamma) + (h_c + A\theta) + \frac{2c_1}{\lambda^2} (K_3(a-\theta) - \frac{b_3}{a-\theta} - b_2 s)] (e^{-(a-\theta)T} - 1),$$

$$Y_3 = \frac{(h_c + A\theta)}{(a-\theta-\gamma)} (1 - e^{(a-\theta-\gamma)T}) \text{ and}$$

$$Y_4 = \frac{1 - e^{-\gamma T}}{\gamma} [s(b_1 - b_2 s) + (\frac{s\lambda - c_2}{\lambda}) (K_3(a-\theta) - \frac{b_3}{a-\theta} - (b_1 - b_2 s))] + \frac{c_1}{\lambda^2} [(K_3(a-\theta) - \frac{b_3}{a-\theta} - (b_1 - b_2 s)) - (h_c + A\theta)K_3 - c_3] + \frac{b_3}{\gamma^2} (1 - e^{-\gamma T} - \gamma T e^{-\gamma T}) (1 - \frac{h_c + A\theta}{a-\theta})$$

Here, η is a function of S . To find out maximum value of η , we must first calculate critical point S^* using the equation $\frac{d\eta}{dS} = 0$ and then $\frac{d^2\eta}{dS^2}$ must have negative value at $S = S^*$.

$$\begin{aligned} \frac{d\eta}{dS} &= \frac{dK_1}{dS} Y_1 + \frac{dK_2}{dS} Y_2 + \frac{dK_3}{dS} Y_3 = 0 \\ \Rightarrow 2K_1(S) \left(\frac{e^{(a-\theta)T}}{e^{(a-\theta)T} - e^{-(a-\theta-\gamma)T}} \right) Y_1 + \left(\frac{e^{(a-\theta)T}}{e^{(a-\theta)T} - e^{-(a-\theta-\gamma)T}} \right) Y_2 + \\ &\left(\frac{e^{-(a-\theta-\gamma)T}}{e^{-(a-\theta-\gamma)T} - e^{(a-\theta)T}} \right) Y_3 = 0 \end{aligned} \tag{18}$$

Solving above for optimal value of S , we get

$$\begin{aligned} S^* &= \frac{1}{\frac{2Y_1 e^{2(a-\theta)T}}{e^{(a-\theta)T} - e^{-(a-\theta-\gamma)T}} - \frac{Y_2 (e^{(a-\theta)T} - e^{-(a-\theta-\gamma)T})}{e^{(a-\theta)T} Y_3 - e^{-(a-\theta-\gamma)T} Y_2} - \frac{1}{K_1 (1 - e^{(a-\theta)T})} - \frac{1}{a - \theta}} \end{aligned} \tag{19}$$

And

$$\begin{aligned} \frac{d^2\eta}{dS^2} &= \frac{2Y_1 \left[\frac{e^{(a-\theta)T}}{e^{(a-\theta)T} - e^{-(a-\theta-\gamma)T}} \right]}{-2c_1 [2(a - \theta) - \gamma] [1 - e^{-(2(a-\theta)-\gamma)T}]} \parallel \frac{e^{(a-\theta)T}}{e^{-(a-\theta-\gamma)T} - e^{(a-\theta)T}} < 0 \text{ as } [a > (\theta + \gamma)] \end{aligned}$$

Hence S^* is the critical point at which η attains its maximum value. Putting S^* in EQ.(18), we have

$$\eta_{max} = \eta(S^*) = (K_1(S^*))^2 Y_1 + K_1(S^*) Y_2 + K_2(S^*) Y_3 + Y_4 \tag{20}$$

In next section, numerical example is given to prove the validity of the model.

Table 1. Effects of Selling Price on Economic Replenishment Quantity and Pro T

s	K_1	K_2	K_3	Y_1	Y_2	Y_3	Y_4	S^*	η
5	172.2232	-604.5961	507.2	-0.09266	18.43775	-16.13687	-10404.07	74.82707	-220.7472
10	172.2232	-512.3027	405.2	-0.09266	18.43775	-16.13687	-8551.058	65.12049	142.9478
15	172.2232	-420.0093	303.2	-0.09266	18.43775	-16.13687	-6802.997	55.41391	401.6591
20	172.2232	-327.7158	201.2	-0.09266	18.43775	-16.13687	-5159.883	45.70732	555.4449
25	172.2232	-235.4224	99.2	-0.09266	18.43775	-16.13687	-3621.716	36.00074	604.2853
30	172.2232	-143.129	-2.8	-0.09266	18.43775	-16.13687	-2188.496	26.29416	548.1787
35	172.2232	-50.83558	-104.8	-0.09266	18.43775	-16.13687	-860.2228	16.58757	387.125
40	172.2232	41.45782	-206.8	-0.09266	18.43775	-16.13687	363.1032	6.88099	121.1244
45	172.2232	133.7512	-308.8	-0.09266	18.43775	-16.13687	1481.482	-2.825593	-249.8231

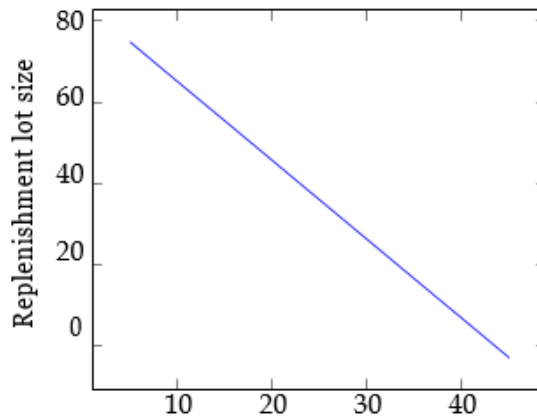


Figure 1. Selling Price Per Unit

Numerical Example

We consider an inventory model with following parameters in appropriate units: $\alpha = 0.2$, $\theta=0.1$, $r=0.16$ units, $i=0.14$ units, $\lambda=0.4$, $hc=0.5$ currency units, $c1=0.5$ currency units, $c2=0.2$ currency units, $c3=25$ currency units, $b1=90$, $b2=2.2$, $b3=0.2$, $s=30$ units, $A=150$ units and $T=1$ year. Then using equation (19) and (20), the optimal value for initial lot size is $S^*=26.29416$ units and maximum pro t, $\eta_{max}=548.1787$ currency units.

Observations

It is observed that with the increase in selling price, demand of the product declines which forces a business firm to maintain reduced inventory level. Figure 1 illustrates the relationship between Selling price per unit and Replenishment lot size to be maintained by the firm. The negative slope of the curve shows fall in stock level with increase in per unit selling price.

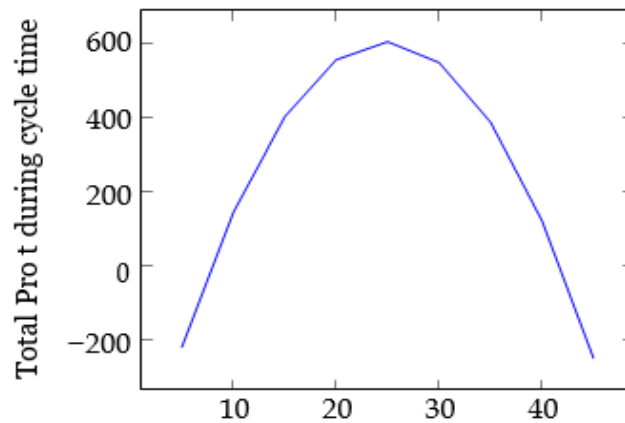


Figure 2. Selling Price Per Unit

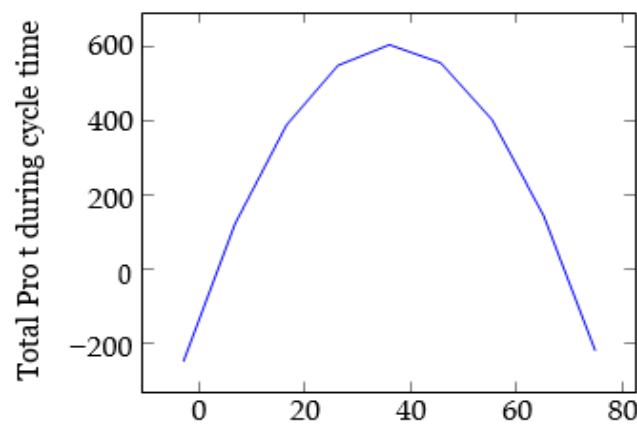


Figure 3. Replenishment lot Size

Table 1 provides the effects of increase in selling price per unit on the total profit during cycle time increases. Initially, total profit rises after that for the further rise in prices, total profit falls resulting in the convex shaped graph.

Figure 2 represents the relationship between selling price per unit and the total profit during cycle time. The point at which the slope of the curve changes its sign from positive to negative represents the optimal selling price of the product for which rm will earn maximum profit.

Table 1 also depicts the relation between replenishment lot size and total profit during cycle time. With the decrease in optimum initial stock level, total profit follows a convex curve showing increase in initial stages and a downfall in later stage.

Figure 3 outlines the relationship between optimum initial inventory level and total profit during cycle time. The point at which the slope of the curve changes its sign from positive to negative represents the optimal stock level to be maintained by the rm to earn maximum profit.

This study finds out optimum stock level to be maintained to gain maximum profits with minimum selling price considering various other parameters such as holding cost, deterioration and marketing efforts.

CONCLUSION

This model discusses a perfect market situation of modern times where demand is dependent upon selling price, time and marketing efforts. Marketing efforts help a seller to create interest in the product and add more visibility of the product which in return create demand. The advanced technology and social media provides an inexpensive way to reach the potential customers. Keeping in mind the demand rate, holding cost of inventory, selling price per unit, cost of marketing efforts and other fixed cost, strategy is designed to fulfill the objective of the organisation which is profit maximization. Various parameters help in deciding the optimum inventory levels to be maintained by the firm to gain maximum profits which are explained through numerical example.

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